Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2342649388

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 1 **(S1)** **October/November 2019**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

This document consists of **14** printed pages and **2** blank pages.

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1 Twelve tourists were asked to estimate the height, in metres, of a new building. Their estimates were as follows.

50 45 62 30 40 55 110 38 52 60 55 40

(i) Find the median and the interquartile range for the data. [3]

For median, arrange them in ascending order

30 38 40 40 45 50 52 55 55 60 62 110

Median = $\frac{50+52}{2} = 51$

LQ Median UQ

30 38 40 | 40 45 50 | 52 55 55 | 60 62 110

U.Q = 57.5

L.Q = 40

I.Q = 57.5 − 40 = 17.5

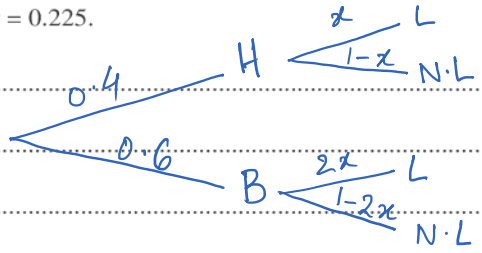
(ii) Give a disadvantage of using the mean as a measure of the central tendency in this case. [1]

110 which is an extreme value affects results

2 Benju cycles to work each morning and he has two possible routes. He chooses the hilly route with probability 0.4 and the busy route with probability 0.6. If he chooses the hilly route, the probability that he will be late for work is x and if he chooses the busy route the probability that he will be late for work is $2x$. The probability that Benju is late for work on any day is 0.36.

(i) Show that $x = 0.225$. [2]

(H & L) or (B & L)



$(0.4 \times x) + (0.6 \times 2x) = 0.36$

$1.6x = 0.36$

$x = 0.225$

Conditional probability

(ii) Given that Benju is not late for work, find the probability that he chooses the hilly route. [3]

$P(H \mid N.L) = \dfrac{P(H \cap N.L)}{P(N.L)}$

Probability that Benju took hilly route & also doesn't get late is $0.4 \times (1-0.225) = 0.31$

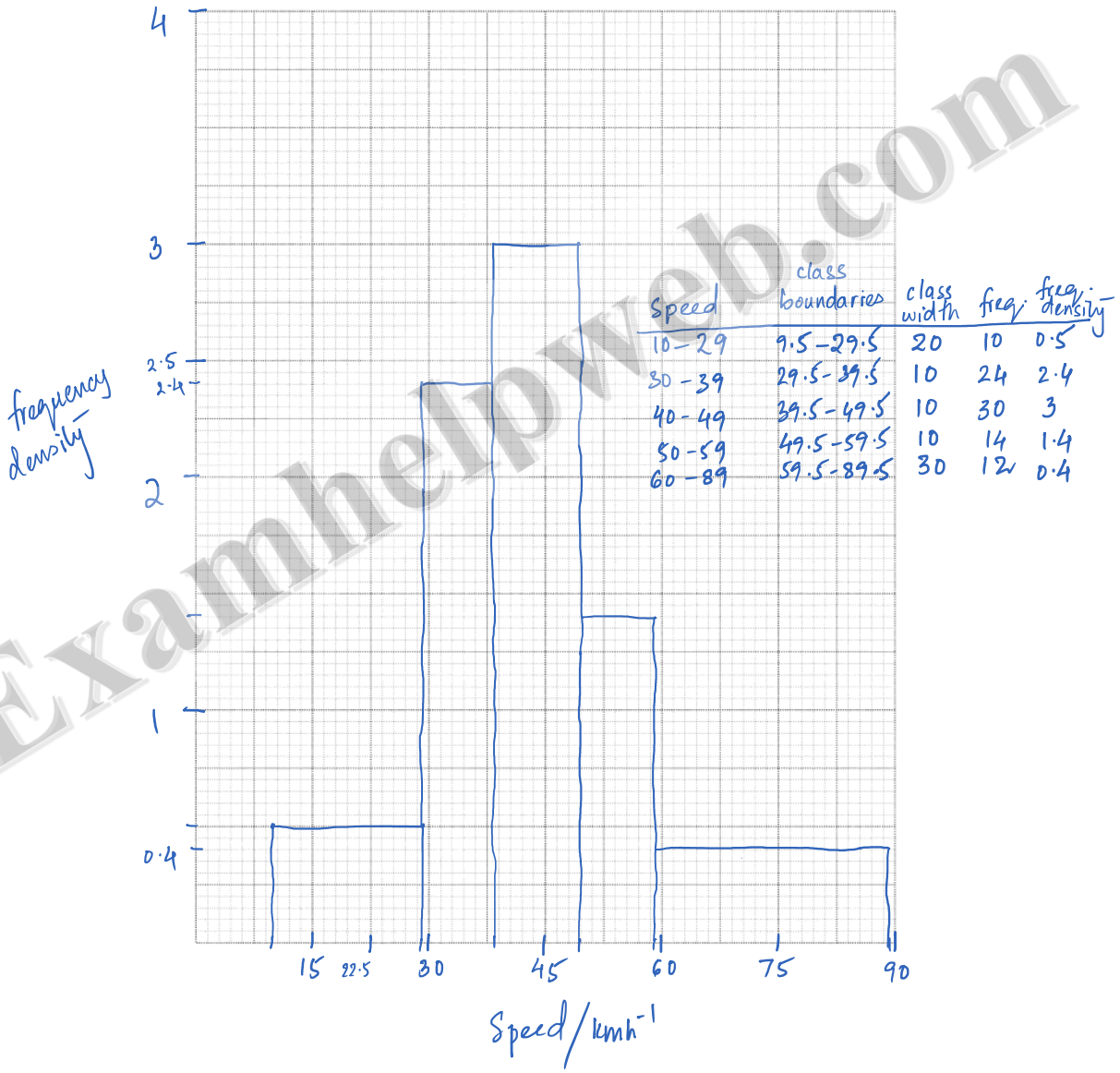
Probability that Benju does not get late is $1 - 0.36 = 0.64$

$= \dfrac{0.31}{0.64} = 0.484$

3 The speeds, in km h^{-1}, of 90 cars as they passed a certain marker on a road were recorded, correct to the nearest km h^{-1}. The results are summarised in the following table.

Speed (km h^{-1})	10 – 29	30 – 39	40 – 49	50 – 59	60 – 89
Frequency	10	24	30	14	12

(i) On the grid, draw a histogram to illustrate the data in the table. [4]



Speed	class boundaries	class width	freq.	freq. density
10 – 29	9·5 – 29·5	20	10	0·5
30 – 39	29·5 – 39·5	10	24	2·4
40 – 49	39·5 – 49·5	10	30	3
50 – 59	49·5 – 59·5	10	14	1·4
60 – 89	59·5 – 89·5	30	12	0·4

(ii) Calculate an estimate for the mean speed of these 90 cars as they pass the marker. [2]

Speed	midvalue (x)	(f)	$f(x)$
10 – 29	19.5	10	195
30 – 39	34.5	24	828
40 – 49	44.5	30	1335
50 – 59	54.5	14	763
60 – 89	74.5	12	894
			$\Sigma fx = 4015$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{4015}{90} = 44.6$$

Since n is not too large, we can use binomial distribution

4 In Quarendon, 66% of households are satisfied with the speed of their wifi connection.

n = 10 P(X ≥ 8)

(i) Find the probability that, out of 10 households chosen at random in Quarendon, at least 8 are satisfied with the speed of their wifi connection. [3]

n = 10, r = 8, 9 & 10, success, p = 0.66, q = 0.34

P(X = 8) → 10C8 × (0.66)^8 × (0.34)^2
P(X = 9) → 10C9 × (0.66)^9 × (0.34)^1
P(X = 10) → 10C10 × (0.66)^10 × (0.34)^0
= 0.284

(ii) A random sample of 150 households in Quarendon is chosen. Use a suitable approximation to find the probability that more than 84 are satisfied with the speed of their wifi connection. [5]

normal distribution

$P(X \geqslant 84) = ?$ $\quad n = 150, \; p = 0.66, \; q = 0.34$

greater than 84 so for z it should be 84.5

$\mu = 150 \times 0.66 = 99 \qquad \sigma^2 = 150 \times 0.66 \times 0.34 = 33.66$

Standardise X to z

$$z = \frac{84.5 - \mu}{\sigma} = \frac{84.5 - 99}{\sqrt{33.66}} = -2.499$$

$P(X > 84) \approx P(z > -2.499)$

$P(z < 2.499)$

$\Phi(2.499) = 0.994$

5 A fair red spinner has four sides, numbered 1, 2, 3, 3. A fair blue spinner has three sides, numbered −1, 0, 2. When a spinner is spun, the score is the number on the side on which it lands. The spinners are spun at the same time. The random variable X denotes the score on the red spinner minus the score on the blue spinner.

(i) Draw up the probability distribution table for X. [4]

Possible scores are 2, 1, −1, 3, 0, 4

x	−1	0	1	2	3	4
$P(X=x)$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{6}$
	1−2	2−2	1−0 3−2	1−(−1) 2−0	2−(−1) 3−0	3−(−1)

(ii) Find Var(X). [3]

$E(X^2) = \frac{71}{12}$

$E(X) = \frac{23}{12}$

$\text{Var}(X) = E(X^2) - (E(X))^2$

$= 2 \cdot 24$

6 The heights, in metres, of fir trees in a large forest have a normal distribution with mean 40 and standard deviation 8.

(i) Find the probability that a fir tree chosen at random in this forest has a height less than 45 metres. [2]

$P(X<45)$ Standardise X to z

$z = \frac{45-40}{8} = 0.625$

$P(X<45) \approx P(z<0.625)$

$\Phi(0.625) = 0.734$

(ii) Find the probability that a fir tree chosen at random in this forest has a height within 5 metres of the mean. [2]

5 meters less or greater than mean

$P(35<X<45)$

$z = \frac{35-40}{8} = -0.625$ & $z = \frac{45-40}{8} = 0.625$

$P(-0.625<z<0.625) = \Phi 0.625 + \Phi(0.625) - 1$

$= 0.734 + 0.734 - 1$

$= 0.468$

In another forest, the heights of another type of fir tree are modelled by a normal distribution. A scientist measures the heights of 500 randomly chosen trees of this type. He finds that 48 trees are less than 10 m high and 76 trees are more than 24 m high.

(iii) Find the mean and standard deviation of the heights of trees of this type. [5]

Probability that trees are higher than 24m $= \frac{76}{500} = 0.152$

Probability that trees are less than 10m high $= \frac{48}{500} = 0.096$

$P(X > 24) = 0.152$ & $P(X < 10) = 0.096$

$z = \frac{24-\mu}{\sigma}$ $\quad z = \frac{10-\mu}{\sigma}$

$P\left(z > \frac{24-\mu}{\sigma}\right) = 0.152$ $\quad P\left(z < \frac{10-\mu}{\sigma}\right) = 0.096$

$1 - \Phi\left(\frac{24-\mu}{\sigma}\right) = 0.152$ $\quad \Phi\left(\frac{10-\mu}{\sigma}\right) = 0.096$

$\frac{24-\mu}{\sigma} = \Phi^{-1}(0.848)$ $\quad \frac{10-\mu}{\sigma} = \Phi(0.096)$

$\frac{24-\mu}{\sigma} = 1.028$ $\quad \frac{10-\mu}{\sigma} = -1.305$

$24 - 1.028\sigma = 10 + 1.305\sigma$

$\sigma = 6$ & $\mu = 17.83$

7 **(i)** Find the number of different ways in which the 9 letters of the word TOADSTOOL can be arranged so that all three Os are together and both Ts are together. [1]

What alphabets are left ? ADSL

6! → _OOO_TT_ _ _ _ 6 places available

720 ways

_O_A_D_S_O_O_L_

(ii) Find the number of different ways in which the 9 letters of the word TOADSTOOL can be arranged so that the Ts are not together. [4]

No. of ways Ts are next to eachother = $\frac{8! \times 2!}{2! \times 3!} = 6720$ (8! : places available; 2! : no. of Ts; 2Ts → 2!, 3Os → 3!)

No. of ways of arranging 9 letters = $\frac{9!}{2! \times 3!} = 30240$ spaces

No. of ways Ts are not together = 30240 − 6720

= 23520

(iii) Find the probability that a randomly chosen arrangement of the 9 letters of the word TOADSTOOL has a T at the beginning and a T at the end. [2]

T _ _ _ _ _ _ _ I

OADSOOL

7 spaces available & 7 letters to fill → $\frac{7!}{3!}$ = 840

3! ← Among 7 letters that will fit in spaces are 3 Os

Probability = $\frac{840}{30240}$ = $\frac{1}{36}$

(iv) Five letters are selected from the 9 letters of the word TOADSTOOL. Find the number of different selections if the five letters include at least 2 Os and at least 1 T. [4]

combination

OOT _ _ ADSL are the choices so 4C2 & 2 spaces

OOTT _ ADSL are the choices so 4C1

OOOT _ ADSL are the choices so 4C1

OOOTT No more choices so this is one selection

4C2 + 4C1 + 4C1 + 1 = 15 ways

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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