Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8279021580

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 The coefficient of x^3 in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant k. [4]

$(1+kx)(1-10x+40x^2-80x^3)$

$-80x^3+40kx^3$

$-80+40k=20$

$k=\frac{5}{2}$

2 The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where p is positive.

Find the sum to infinity of the progression. [5]

$a_1 = 2p+6$, $a_2 = -2p$, $a_3 = p+2$, $S_\infty = ?$

$r = \frac{-2p}{2p+6} = \frac{p+2}{-2p} \longrightarrow 4p^2 = 2p^2 + 10p + 12$

$2p^2 - 10p - 12 = 0$ so $p = 6$

$S_\infty = \frac{a_1}{1-r} = \frac{2(6)+6}{1-(-2/3)} = \frac{54}{5}$

3 The equation of a curve is $y = 2x^2 + m(2x + 1)$, where m is a constant, and the equation of a line is $y = 6x + 4$.

they have 2 roots, use $b^2-4ac>0$

Show that, for all values of m, the line intersects the curve at two distinct points. [5]

$2x^2+2mx+m = 6x+4$

$2x^2+2mx-6x+m-4=0$

$a=2$, $b=2m-6$, $c=m-4$

$(2m-6)^2-4(2)(m-4)>0$

$4m^2-24m+36-8m+32>0$

$4m^2-32m+68>0$

$m^2-8m+17>0$

$\left(m-\frac{8}{2}\right)^2-\frac{1}{4}\times(-8)^2+17>0$

$(m-4)^2+1>0$ for all m values

4 The sum, S_n, of the first n terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The kth term in the progression is greater than 200. $S_n = \frac{n}{2}(2a_1 + d(n-1))$

Find the smallest possible value of k. [5]

$S_1 = 1^2 + 4(1) = 5$ so $a_1 = 5$

$S_2 = 2^2 + 4(2) = 12$ so $12 = \frac{2}{2}(2(5) + d(2-1))$

$d = 2$

$a_n = a_1 + d(n-1)$

$5 + 2(k-1) > 200$

$5 + 2k - 2 > 200$

$k > 98.5 \therefore k = 99$

5 Functions f and g are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$

$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

(a) Find the value of fg(7). [1]

$g(7) = \frac{4}{7+1} = 0.5$

$f(0.5) = 4(0.5) - 2 = 0$

(b) Find the values of x for which $f^{-1}(x) = g^{-1}(x)$. [5]

$y = 4x - 2 \longrightarrow \frac{y+2}{4} = x \longrightarrow f^{-1}(x) = \frac{x+2}{4}$

$y = \frac{4}{x+1} \longrightarrow \frac{4}{y} - 1 = x \longrightarrow g^{-1}(x) = \frac{4}{x} - 1$

$\frac{x+2}{4} = \frac{4-x}{x}$

$x^2 + 2x = 16 - 4x$

$x^2 + 6x - 16 = 0$

$x = 2, -8$

6 **(a)** Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

$$\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x} + 1\right)$$

$$\left(\frac{1-\sin x}{\cos x}\right)\left(\frac{1+\sin x}{\sin x}\right)$$

$$\frac{1-\sin^2 x}{\cos x \sin x} = \frac{\cos^{\cancel{2}} x}{\cancel{\cos x}\sin x} = \frac{\cos x}{\sin x} \equiv \frac{1}{\tan x}$$

(b) Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

$$\frac{1}{\tan x} = 2\tan^2 x$$

$$\tan^3 x = \frac{1}{2}$$

$$\tan x = 0.7937$$

$$x = 38.44°$$

7 The point (4, 7) lies on the curve $y = f(x)$ and it is given that $f'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

$\frac{dy}{dx}$

$\frac{dx}{dt} = 0.12$

(a) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second.

$\frac{dy}{dt} = ?$

Find the rate of increase of the y-coordinate when $x = 4$. [3]

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$\frac{dy}{dx} \text{ at } x = 4 \longrightarrow 6(4)^{-\frac{1}{2}} - 4(4)^{-\frac{3}{2}} = \frac{5}{2}$$

$$\frac{dy}{dt} = \frac{5}{2} \times 0.12 = 0.3$$

(b) Find the equation of the curve. [4]

$$\frac{dy}{dx} = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}} \qquad (4, 7)$$

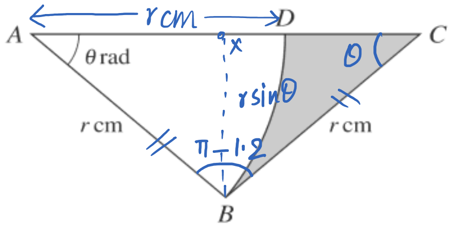
$$\int 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}} \longrightarrow 6\left[2x^{\frac{1}{2}}\right] - 4\left[-2x^{-\frac{1}{2}}\right] + c$$

$$12x^{\frac{1}{2}} + 8x^{-\frac{1}{2}} + c = y$$

$$12(4)^{\frac{1}{2}} + 8(4)^{-\frac{1}{2}} + c = 7 \quad \text{so} \quad c = -21$$

$$y = 12x^{\frac{1}{2}} + \frac{8}{x^{\frac{1}{2}}} - 21$$

8



In the diagram, ABC is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A.

(a) Express the area of the shaded region in terms of r and θ. [3]

$BX = r\sin\theta \qquad AX = r\cos\theta$

Area of triangle $ABC = 2\left(\frac{1}{2} \times r\cos\theta \times r\sin\theta\right)$

$= r^2\cos\theta\sin\theta$

A of sector $ABD = \frac{1}{2}r^2\theta$

Shaded region $= r^2\sin\theta\cos\theta - \frac{r^2\theta}{2}$

(b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

Arc $BD = 10 \times 0.6 = 6$ $\hat{ABC} = \pi - 1.2$

$AC = \sqrt{10^2 + 10^2 - (2 \times 10 \times 10 \times \cos(\pi - 1.2))} = 16.51$

$CD = 16.51 - 10 = 6.507$ cm

Perimeter $= 6.507 + 10 + 6 = 22.51$

9 A circle has centre at the point $B(5, 1)$. The point $A(-1, -2)$ lies on the circle.

(a) Find the equation of the circle. [3]

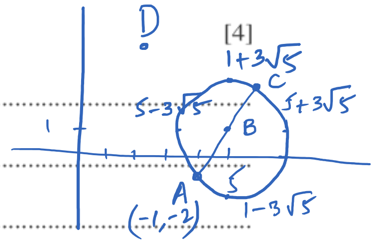
$(x-h)^2 + (y-k)^2 = r^2$ where (h,k) is the centre of a circle & r is its radius

Radius, $r = \sqrt{(-2-1)^2 + (-1-5)^2} = 3\sqrt{5}$

Eq. $\longrightarrow (x-5)^2 + (y-1)^2 = 45$

Point C is such that AC is a diameter of the circle. Point D has coordinates $(5, 16)$.

(b) Show that DC is a tangent to the circle. [4]



$AB = BC = 3\sqrt{5}$

$C(11, 4)$

Gradient of AC $\longrightarrow \dfrac{4+2}{11+1} = \dfrac{6}{12} = \dfrac{1}{2}$

Gradient of CD $\longrightarrow \dfrac{4-16}{11-5} = \dfrac{-12}{6} = -2$

$\dfrac{1}{2} \times -2 = -1$ thus CD is perpendicular to AC

The other tangent from D to the circle touches the circle at E.

(c) Find the coordinates of E. [2]

$DE \longrightarrow y = 2x + 6$

$(x-5)^2 + (2x+6-1)^2 = 45$

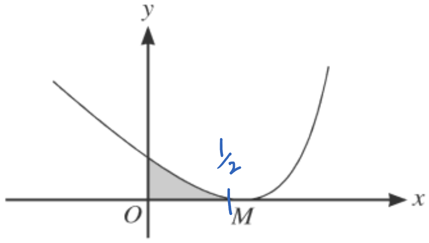
$x^2 - 10x + 25 + 4x^2 + 20x + 25 = 45$

$5x^2 + 10x + 5 = 0$

$x^2 + 2x + 1 = 0$

$E\ (-1, 4)$

10



The diagram shows part of the curve $y = \frac{2}{(3-2x)^2} - x$ and its minimum point M, which lies on the x-axis.

(a) Find expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\int y\,dx$. [6]

$y = 2(3-2x)^{-2} - x$

$\frac{dy}{dx} = -4(3-2x)^{-3} \times -2 - 1 = 8(3-2x)^{-3} - 1$

$\frac{d^2y}{dx^2} = -24(3-2x)^{-4} \times -2 = 48(3-2x)^{-4}$

$\int 2(3-2x)^{-2} - x\,dx \rightarrow 2\left[\frac{(3-2x)^{-1}}{-1} \times \frac{1}{-2}\right] - \frac{x^2}{2}$

$= \frac{1}{3-2x} - \frac{x^2}{2}$

(b) Find, by calculation, the x-coordinate of M. [2]

$\frac{dy}{dx} = 0$

$\frac{8}{(3-2x)^3} - 1 = 0$

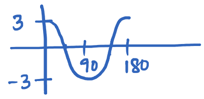
$8 = (3-2x)^3$

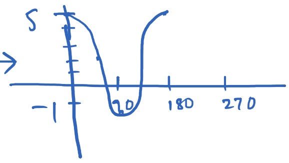
$2 = 3 - 2x \longrightarrow x = \frac{1}{2}$

(c) Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

$\int_0^{\frac{1}{2}} \frac{2}{(3-2x)^2} - x \, dx$

$\left[\frac{1}{3-2x} - \frac{x^2}{2} \right]_0^{\frac{1}{2}} = \frac{3}{8} - \frac{1}{3} = \frac{1}{24}$

$3\cos 2x \longrightarrow$ 

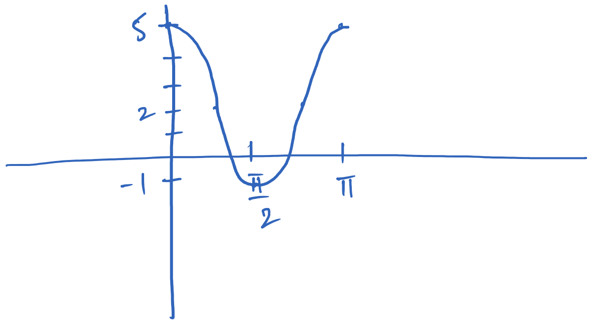
$3\cos 2x + 2 \longrightarrow$ 

11 A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

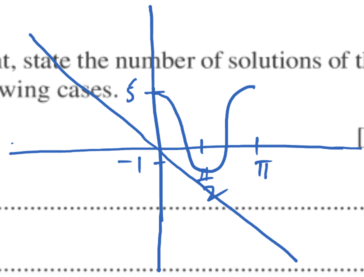
(a) State the greatest and least values of y. [2]

$-1 < y < 5$

(b) Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]



(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

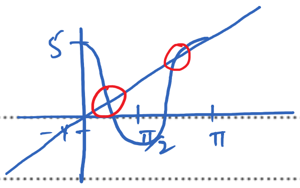


(i) $k = -3$ [1]

$3\cos 2x + 2 = -3x$

No solution

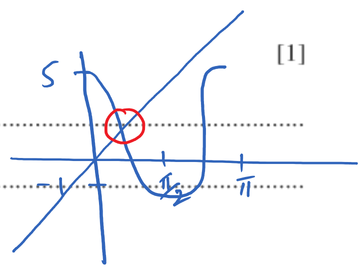
(ii) $k = 1$ [1]



$3\cos 2x + 2 = x$

2 solutions

(iii) $k = 3$ [1]



$3\cos 2x + 2 = 3x$

1 solution

Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3\cos 2x + 2,$$
$$g(x) = f(2x) + 4,$$
$$h(x) = 2f\left(x + \tfrac{1}{2}\pi\right).$$

(d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

$g(x) = f(2x) + 4$ ← moves 4 units up

graph shrinks by 2 along x-axis

Strech by factor of $\frac{1}{2}$ in x-direction

Translation $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

(e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

$h(x) = 2f\left(x + \frac{\pi}{2}\right)$

Streches 2 times along y-axis

moves $\frac{\pi}{2}$ to the left

Stretch by a factor of 2 along y-axis

Translation $\begin{pmatrix} -\frac{\pi}{2} \\ 0 \end{pmatrix}$

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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