Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 5 1 9 7 4 5 9 1 5 2 *

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **February/March 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

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[Turn over

1 Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \sqrt{(1 - \tan x)}\, \mathrm{d}x,$$

giving your answer correct to 3 decimal places. [3]

$$\int_a^b y\, dx \longrightarrow \frac{1}{2}h\left[y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n\right]$$

where $h = \dfrac{b-a}{n}$ ← no. of intervals

$n = 3$ so $h = \dfrac{\pi/4 - 0}{3} = \dfrac{\pi}{12}$

There are 3 intervals so

$\int$: $\pi/4 \rightarrow y_n$, $\pi/6 \rightarrow y_2$, $\pi/12 \rightarrow y_1$, $0 \rightarrow y_0$

$$\frac{1}{2} \times \frac{\pi}{12}\left[\sqrt{1-\tan y_0} + 2\left(\sqrt{1-\tan y_1} + \sqrt{1-\tan y_2}\right) + \sqrt{1-\tan y_n}\right]$$

$$\frac{\pi}{24}\left[1 + 2\left(0.8556 + 0.65\right) + 0\right] = 0.525$$

2 Expand $\sqrt[4]{(1 - 4x)}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

$(1-4x)^{1/4} \longrightarrow 1 - x - \frac{3}{2}x^2 - \frac{7}{2}x^3$

3 **(i)** Using the expansions of $\cos(3x + x)$ and $\cos(3x - x)$, show that

$$\tfrac{1}{2}(\cos 4x + \cos 2x) \equiv \cos 3x \cos x.$$ [3]

$\cos(3x+x) = \cos 3x \cos x - \sin 3x \sin x$

$\cos(3x-x) = \cos 3x \cos x + \sin 3x \sin x$

$\frac{1}{2}(\cos 3x \cos x - \cancel{\sin 3x \sin x} + \cos 3x \cos x + \cancel{\sin 3x \sin x})$

$\frac{1}{\cancel{2}}(\cancel{2}\cos 3x \cos x) = \cos 3x \cos x$

(ii) Hence show that $\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \cos 3x \cos x \, dx = \frac{3}{8}\sqrt{3}$. [3]

$\frac{1}{2}\int_{-\pi/6}^{\pi/6} \cos 4x + \cos 2x \longrightarrow \frac{1}{2}\left[\frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right]$

$\frac{\sin 4x}{8} + \frac{\sin 2x}{4} \longrightarrow \left[\frac{\sin 4x + 2\sin 2x}{8}\right]_{-\pi/6}^{\pi/6}$

$\frac{\sqrt{3}/2 + \sqrt{3}}{8} - \left(\frac{-\sqrt{3}/2 - \sqrt{3}}{8}\right) \longrightarrow \frac{3\sqrt{3}}{8}$

4 The variables x and y satisfy the equation $y^n = Ax^3$, where n and A are constants. It is given that $y = 2.58$ when $x = 1.20$, and $y = 9.49$ when $x = 2.51$.

(i) Explain why the graph of $\ln y$ against $\ln x$ is a straight line. [2]

$y^n = Ax^3 \longrightarrow n \ln y = \ln Ax^3 \longrightarrow n \ln y = \ln A + 3\ln x$

$n \ln y = \ln A + 3\ln x$ is in the form of $y = mx + c$ which is a linear equation where $Y = \ln y$ and $X = \ln x$ thus its a straight line

(ii) Find the values of n and A, giving your answers correct to 2 decimal places. [4]

$n \ln y = 3\ln x + \ln A$

$n \ln 2.58 = 3\ln 1.2 + \ln A$ so $\ln A = n \ln 2.58 - 3\ln 1.2 \rightarrow 1$

$n \ln 9.49 = 3\ln 2.51 + \ln A$ so $\ln A = n \ln 9.49 - 3\ln 2.51 \rightarrow 2$

$n \ln 2.58 - 3\ln 1.2 = n \ln 9.49 - 3\ln 2.51$

$3\ln 2.51 - 3\ln 1.2 = n\ln 9.49 - n\ln 2.58$

$3\ln\left(\frac{2.51}{1.2}\right) = n\ln\left(\frac{9.49}{2.58}\right)$

$n = 1.70$ & $A = 2.90$

5 The parametric equations of a curve are

$$x = 2t + \sin 2t, \quad y = 1 - 2\cos 2t,$$

for $-\frac{1}{2}\pi < t < \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = 2\tan t$. [5]

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

$\frac{dx}{dt} = 2 + 2\cos 2t$ $\quad\quad \frac{dy}{dt} = -2(-\sin 2t \times 2)$

$\frac{dy}{dx} = \frac{4\sin 2t}{2 + 2\cos 2t} = \frac{8\sin t\cos t}{2 + 2(2\cos^2 t - 1)} = \frac{\cancel{8}^{2}\sin t\,\cancel{\cos t}}{\cancel{4}\cos^{\cancel{2}} t}$

$= \frac{2\sin t}{\cos t}$

$= 2\tan t$

(ii) Hence find the x-coordinate of the point on the curve at which the gradient of the normal is 2. Give your answer correct to 3 significant figures. [2]

Gradient of normal $= -1/$gradient of curve

Gradient of curve $= -\frac{1}{2}$ at x-coordinate

Gradient of a curve is its $\frac{dy}{dx}(2\tan x)$

$$2\tan x = -\frac{1}{2}$$

$$x = \tan^{-1}\left(-\frac{1}{4}\right)$$

$$x = 2.90$$

6 The variables x and θ satisfy the differential equation

$$x\cos^2\theta\frac{dx}{d\theta} = 2\tan\theta + 1,$$

for $0 \leqslant \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 1$ when $\theta = \frac{1}{4}\pi$.

(i) Show that $\frac{d}{d\theta}(\tan^2\theta) = \frac{2\tan\theta}{\cos^2\theta}$. [1]

$$\frac{d\tan^2\theta}{d\theta} = 2\tan\theta \times \sec^2\theta$$

$$= \frac{2\tan\theta}{\cos^2\theta}$$

(ii) Solve the differential equation and calculate the value of x when $\theta = \frac{1}{3}\pi$, giving your answer correct to 3 significant figures. [7]

$$\int x\,dx = \int \frac{2\tan\theta + 1}{\cos^2\theta}\,d\theta$$

$$\frac{x^2}{2} = \int \frac{2\tan\theta}{\cos^2\theta} + \frac{1}{\cos^2\theta}$$

$$\frac{x^2}{2} = 2\int \tan\theta\sec^2\theta + \int \sec^2\theta$$

part(i)

$$\frac{x^2}{2} = \tan^2\theta + \tan\theta + c$$

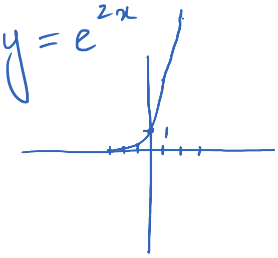
$$\frac{1^2}{2} = \tan^2\frac{\pi}{4} + \tan\frac{\pi}{4} + c \quad \text{so} \quad c = \frac{-3}{2}$$

$$\frac{x^2}{2} = \tan^2\theta + \tan\theta - \frac{3}{2}$$

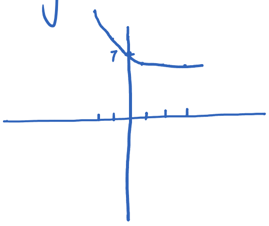
When $\theta = \frac{\pi}{3}$, $x = 2.54$

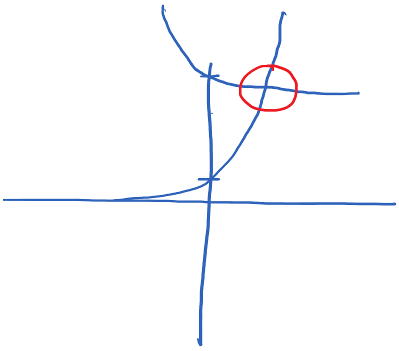
7 **(i)** By sketching suitable graphs, show that the equation $e^{2x} = 6 + e^{-x}$ has exactly one real root. [2]

$y = e^{2x}$



$y = 6 + e^{-x}$





(ii) Verify by calculation that this root lies between 0.5 and 1. [2]

$y = e^{2x} - 6 - e^{-x}$

When $x = 0.5$, $y = -3.888$

When $x = 1$, $y = 1.0212$

One is $-$ve & the other is $+$ve proves that root lies between $x = 0.5$ & $x = 1$

(iii) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{3}\ln\left(1 + 6e^{x_n}\right)$$

converges, then it converges to the root of the equation in part **(i)**. [2]

$$e^{2x} = 6 + e^{-x}$$

$$e^{2x} = 6 + \frac{1}{e^x}$$

$$e^{2x} = \frac{6e^x + 1}{e^x}$$

$$e^{3x} = 6e^x + 1$$

$$3x \ln e = \ln\left(6e^x + 1\right)$$

$$x = \frac{\ln\left(6e^x + 1\right)}{3}$$

(iv) Use this iterative formula to calculate the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

$$x_{n+1} = \frac{\ln\left(6e^{x_n} + 1\right)}{3} \qquad x_1 = 1.021177$$

$$x_{1+1} = \frac{\ln\left(6e^{x_1} + 1\right)}{3} \rightarrow x_2 = 0.95708$$

$$x_{2+1} = \frac{\ln\left(6e^{x_2} + 1\right)}{3} \rightarrow x_3 = 0.93696$$

$$x_{3+1} = \frac{\ln\left(6e^{x_3} + 1\right)}{3} \rightarrow x_4 = 0.93066$$

$$x_{4+1} = \frac{\ln\left(6e^{x_4} + 1\right)}{3} \rightarrow x_5 = 0.92869$$

$$x_{5+1} = \frac{\ln\left(6e^{x_5} + 1\right)}{3} \rightarrow x_6 = 0.92807$$

$$x_7 = 0.92788$$

$$x_8 = 0.92782$$

$$x_9 = 0.92780$$

$$\therefore \quad x = 0.928$$

8 Let $f(x) = \dfrac{5x^2 + x + 27}{(2x+1)(x^2+9)}$ → its numerator will be having x'

(i) Express $f(x)$ in partial fractions. [5]

$$\frac{A}{2x+1} + \frac{Bx+C}{x^2+9} \equiv 5x^2+x+27$$

$$A(x^2+9) + (Bx+C)(2x+1) \equiv 5x^2+x+27$$

Lets compare coefficients of x^2

$$A + 2B = 5 \longrightarrow A = 5 - 2B \longrightarrow (1)$$

Lets compare coefficients of x

$$B + 2C = 1 \longrightarrow (2)$$

Lets compare constants

$$9A + C = 27 \longrightarrow (3)$$

(1) into (3)

$$9(5-2B) + C = 27 \longrightarrow 18B - C = 18 \longrightarrow (4)$$

$$(2) = (4)$$

$$\frac{1-B}{2} = 18B - 18 \longrightarrow 1 - B = 36B - 36$$

$$B = 1,\ C = 0\ \&\ A = 3$$

(ii) Hence find $\int_0^4 f(x)\,dx$, giving your answer in the form $\ln c$, where c is an integer. [5]

$$\int_0^4 \frac{3}{2x+1} + \frac{x}{x^2+9}\,dx$$

$$\frac{3}{2}\ln(2x+1)$$

Suppose $u = x^2+9$ then

$\frac{du}{dx} = 2x$ & $x\,dx = \frac{du}{2}$

$$\frac{x\,dx}{x^2+9} \longrightarrow \int \frac{1}{u} \times \frac{du}{2}$$

$$\frac{1}{2}\int \frac{1}{u}\,du \longrightarrow \frac{1}{2}\ln u$$

$$= \frac{1}{2}\ln(x^2+9)$$

$$\left[\frac{3}{2}\ln(2x+1) + \frac{1}{2}\ln(x^2+9)\right]_0^4$$

$$[\ln 27 + \ln 5] - [\ln 3] = \ln 27 + \ln\left(\tfrac{5}{3}\right)$$

$$= \ln\left(27 \times \tfrac{5}{3}\right) = \ln 45$$

9 The complex number $1 + 2\mathrm{i}$ is denoted by u.

(i) It is given that u is a root of the equation $2x^3 - x^2 + 4x + k = 0$, where k is a constant.

(a) Showing all working and without using a calculator, find the value of k. [3]

$2(1+2i)^3 - (1+2i)^2 + 4(1+2i) + k = 0$

$2(-11-2i) - (4i-3) + 4 + 8i + k = 0$

$-22 \cancel{-4i} \cancel{-4i} + 3 + 4 \cancel{+8i} + k = 0$

$k = 15$

(b) Showing all working and without using a calculator, find the other two roots of this equation. [4]

Conjugate of u is also one of the roots so

$1+2i$, $1-2i$ & x Now following is the method you can follow to find the 3^{rd} root

Sum of the roots : $1+2i + (1-2i) = 2$

Product of the roots : $(1+2i)(1-2i) = 1 - 4i^2 = 5$

Quadratic factor of the equation constituting these two roots is :

$z^2 - (\text{sum of roots}) + (\text{product of roots})$

$z^2 - 2z + 5$ it means $1+2i$ & $1-2i$ were roots from $z^2 - 2z + 5$

For 3rd root, divide $2x^3-x^2+4x+15=0$ by x^2-2x+5

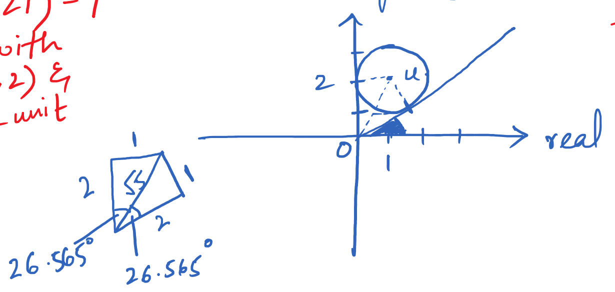
$$
\begin{array}{r}
2x+3 \\
x^2-2x+5 \;\overline{\big)\; 2x^3-x^2+4x+15} \\
-(2x^3-4x^2+10x) \\
\hline
3x^2-6x+15 \\
-(3x^2-6x+15) \\
\hline
0
\end{array}
$$

So the 3rd root is $2x+3=0 \longrightarrow -\frac{3}{2}$

(ii) On an Argand diagram sketch the locus of points representing complex numbers z satisfying the equation $|z-u|=1$. Determine the least value of $\arg z$ for points on this locus. Give your answer in radians correct to 2 decimal places. [4]

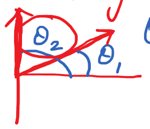
$z-(1+2i)=1$

circle with centre $(1,2)$ & radius 1 unit



Least value of arg z is tangent from origin to the circle's circumference that makes smallest possible angle

Ex.



$\theta_1 < \theta_2$

Shaded part $= 90° - (2\times 26.565°) = 36.87° = 0.64$

10 The line l has equation $\mathbf{r} = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$. The plane p has equation $2x - 3y - z = 4$.

(i) Find the position vector of the point of intersection of l and p. [3]

Eq of line $l \longrightarrow r = \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \longrightarrow r = \begin{pmatrix} 4+\mu \\ 3+2\mu \\ -1-2\mu \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4+\mu \\ 3+2\mu \\ -1-2\mu \end{pmatrix} \longrightarrow 1$$

Plane $p \longrightarrow 2x - 3y - z = 4 \longrightarrow 2$

Put 1 into 2

$$2(4+\mu) - 3(3+2\mu) - (-1-2\mu) = 4$$

$$8 + 2\mu - 9 - 6\mu + 1 + 2\mu = 4 \quad \text{so } \mu = -2$$

Put $\mu = -2$ into l's equation $\begin{pmatrix} 4-2 \\ 3+2(-2) \\ -1-2(-2) \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

(ii) Find the acute angle between l and p. [3]

Normal of plane $p \longrightarrow \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$

Direction vector of line $l \longrightarrow \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

$$\begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \sqrt{2^2 + (-3)^2 + (-1)^2}\sqrt{1^2 + 2^2 + (-2)^2} \cos\theta$$

$$\theta = 10.3^\circ$$

(iii) A second plane q is parallel to l, perpendicular to p and contains the point with position vector $4\mathbf{j} - \mathbf{k}$. Find the equation of q, giving your answer in the form $ax + by + cz = d$. [5]

Plane q contains $\begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix}$ & $r \cdot \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} = 4$ is perpendicular to q

Vector product of l's direction vector & plane p's normal vector

$$\begin{vmatrix} i & j & k \\ 2 & -3 & -1 \\ 1 & 2 & -2 \end{vmatrix} = (6-(-2))i - (-4-(-1))j + (4-(-3))k$$

$$= 8i + 3j + 7k$$

Plane $q \longrightarrow r \cdot \begin{pmatrix} 8 \\ 3 \\ 7 \end{pmatrix} = D$

$D = \begin{pmatrix} 8 \\ 3 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix} = 5$

Plane q contains this position vector

Plane $q \longrightarrow 8x + 3y + 7z = 5$

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.