Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8348568421

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **February/March 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

JC18 03_9709_12/2R

CAMBRIDGE International Examinations

[Turn over

1 A curve passes through the point (4, −6) and has an equation for which $\frac{dy}{dx} = x^{-\frac{1}{2}} - 3$. Find the equation of the curve. [4]

Integration of $\frac{dy}{dx}$ $\int x^{-\frac{1}{2}} - 3\,dx$

$y = 2x^{\frac{1}{2}} - 3x + c$

Use 4, −6 to get value for c

$-6 = 2\sqrt{4} - 3(4) + c$

$c = 2$

$y = 2\sqrt{x} - 3x + 2$

2 **(i)** Find the coefficients of x^2 and x^3 in the expansion of $(1-2x)^7$. [3]

$7C0 \times (1)^7 \times (-2x)^0 + 7C1 \times (1)^6 \times (-2x)^1 + \underbrace{7C2 \times (1)^5 \times (-2x)^2}$

$+ \underbrace{7C3 \times (1)^4 \times (-2x)^3}$

$84x^2 - 280x^3$

(ii) Hence find the coefficient of x^3 in the expansion of $(2+5x)(1-2x)^7$. [2]

$(2+5x)(-14x + 84x^2 - 280x^3)$

$7C1 \times (1)^6 \times (-2x)^1$

$-560x^3 + 420x^3 = -140$

3 On a certain day, the height of a young bamboo plant was found to be 40 cm. After exactly one day its height was found to be 41.2 cm. Two different models are used to predict its height exactly 60 days after it was first measured.

- Model A assumes that the daily amount of growth continues to be constant at the amount found for the first day. Airthmetic progression
- Model B assumes that the daily percentage rate of growth continues to be constant at the percentage rate of growth found for the first day. geometric progression

(i) Using model A, find the predicted height in cm of the bamboo plant exactly 60 days after it was first measured. [2]

Difference $= 1.2$ cm

Total difference $= 1.2 \times 60 = 72$ cm

$72 + 40 = 112$ cm of height aftee 60 days

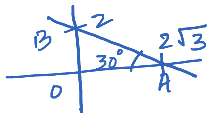
(ii) Using model B, find the predicted height in cm of the bamboo plant exactly 60 days after it was first measured. [3]

Ratio $= \frac{41.2}{40} = 1.03$

$a_{60} = 40 \times (1.03)^{60} = 235.66$ cm

4 A straight line cuts the positive x-axis at A and the positive y-axis at $B\,(0,\ 2)$. Angle $BAO = \frac{1}{6}\pi$ radians, where O is the origin.

(i) Find the exact value of the x-coordinate of A. [2]



$\text{Tan } BAO = \dfrac{OB}{OA}$

$\text{Tan } \frac{\pi}{6} = \dfrac{2}{OA}$ so $OA = 2\sqrt{3}$

$m_1 \times m_2 = -1$

(ii) Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$, where m is given exactly and c is an integer. [4]

Gradient of $AB = \dfrac{2-0}{0-2\sqrt{3}} = -\dfrac{\sqrt{3}}{3}$

Gradient of perpendicular bisector of $AB = \sqrt{3}$

Co-ordinates for perpendicular bisector:

$\dfrac{0+2\sqrt{3}}{2},\ \dfrac{2+0}{2} = \sqrt{3},\ 1$

$y - 1 = \sqrt{3}\,(x - \sqrt{3})$

$y = \sqrt{3}x - 2$

5 (a) Express the equation $\frac{5 + 2\tan x}{3 + 2\tan x} = 1 + \tan x$ as a quadratic equation in $\tan x$ and hence solve the equation for $0 \leqslant x \leqslant \pi$. [4]

$5 + 2\tan x = (1 + \tan x)(3 + 2\tan x)$

$5 + 2\tan x = 3 + 5\tan x + 2\tan^2 x$

$2\tan^2 x + 3\tan x - 2 = 0$

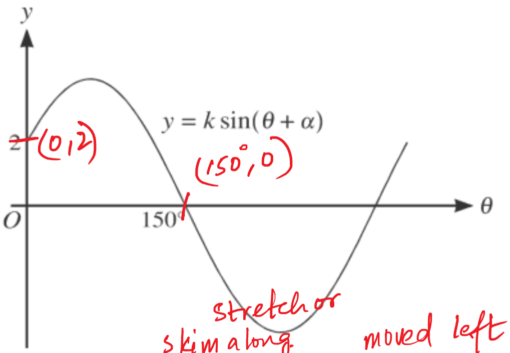
$\tan x = \frac{1}{2}$ $\tan x = -2$

$x = 0.464$ $x = -1.10715$

$\tan x$ is $-$ve in 2^{nd} quadrant in range of $0 \leq x \leq \pi$ so $x = \pi - 1.10715 = 2.034$

$x = 0.464, 2.034$ //

(b)

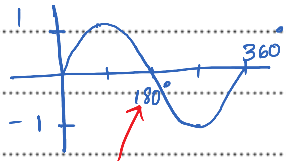


stretch or skim along y-axis

moved left along x-axis

The diagram shows part of the graph of $y = k\sin(\theta + \alpha)$, where k and α are constants and $0° < \alpha < 180°$. Find the value of α and the value of k. [2]

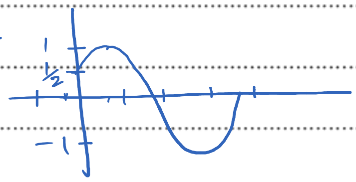
$\sin\theta° \rightarrow$



Here it crosses x-axis at 180° & in the diagram it does at 150° which means the graph moved 30° towards the left

$\alpha = 30°$

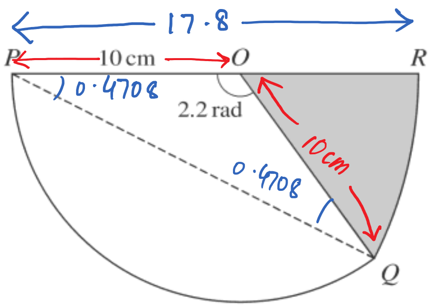
$\sin(\theta + 30°) \rightarrow$



Here it intersects y-axis at $(0, \frac{1}{2})$ & in the diagram above, it was (0,2) so k should be 4.

$k = 4$

6



The diagram shows a sector POQ of a circle of radius 10 cm and centre O. Angle POQ is 2.2 radians. QR is an arc of a circle with centre P and POR is a straight line.

(i) Show that the length of PQ is 17.8 cm, correct to 3 significant figures. [2]

POQ is a sector therefore OQ = OP = 10cm

$PQ = \sqrt{10^2 + 10^2 - (2 \times 10 \times 10 \times \cos 2.2)}$

$= 17.8$ //

(ii) Find the perimeter of the shaded region. [4]

QR is the arc with centre P therefore PR = PQ = 17.8 cm

Arclength QR $= r\theta = 17.8 \times 0.4708 = 8.38$

$8.38 + 10 + (17.8 - 10) = 26.18\text{cm}$

7

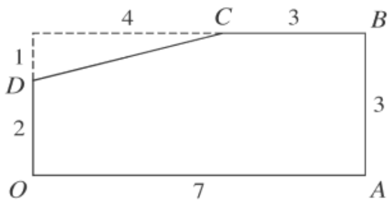


Fig. 1

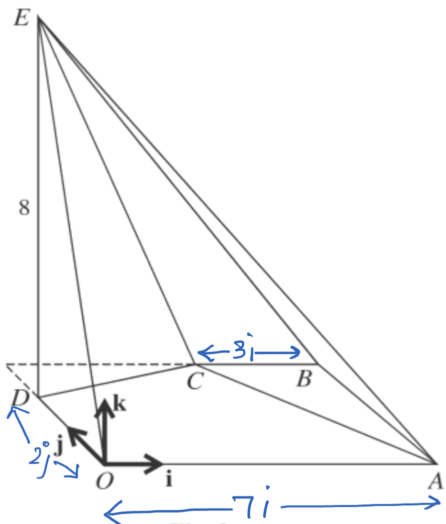


Fig. 2

Fig. 1 shows a rectangle with sides of 7 units and 3 units from which a triangular corner has been removed, leaving a 5-sided polygon $OABCD$. The sides OA, AB, BC and DO have lengths of 7 units, 3 units, 3 units and 2 units respectively. Fig. 2 shows the polygon $OABCD$ forming the horizontal base of a pyramid in which the point E is 8 units vertically above D. Unit vectors **i**, **j** and **k** are parallel to OA, OD and DE respectively.

(i) Find $\overrightarrow{CE}$ and the length of CE. [3]

$\overrightarrow{CE} = \overrightarrow{CA} + \overrightarrow{AO} + \overrightarrow{OD} + \overrightarrow{DE}$

$\overrightarrow{CA} = \overrightarrow{CB} + \overrightarrow{BA} = 3i - 3j$

$\overrightarrow{CE} = 3i - 3j - 7i + 2j + 8k = -4i - j + 8k$

$|\overrightarrow{CE}| = \sqrt{(-4)^2 + (-1)^2 + (8)^2} = 9$

(ii) Use a scalar product to find angle ECA, giving your answer in the form $\cos^{-1}\left(\frac{m}{\sqrt{n}}\right)$, where m and n are integers. [5]

$\overrightarrow{EC} = 4i + j - 8k \quad \overrightarrow{AC} = -3i + 3j$

$$\begin{pmatrix} 4 \\ 1 \\ -8 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix} = \sqrt{4^2 + 1^2 + (-8)^2} \times \sqrt{(-3)^2 + 3^2 + 0^2} \cos\theta$$

$$\frac{-9}{27\sqrt{2}} = \cos\theta \longrightarrow \cos\theta = \frac{-9}{\sqrt{1458}}$$

8 A curve has equation $y = \frac{1}{2}x^2 - 4x^{\frac{3}{2}} + 8x$.

(i) Find the x-coordinates of the stationary points. [5]

$y = \frac{x^2}{2} - 4x^{3/2} + 8x$

$\frac{dy}{dx} = x - 6x^{1/2} + 8$

$x - 6x^{1/2} + 8 = 0$

$x^{1/2} = 2 \quad x^{1/2} = 4$

so $x = 4$ & $x = 16$

(ii) Find $\frac{d^2y}{dx^2}$. [1]

$\frac{dy}{dx} = x - 6x^{\frac{1}{2}} + 8$

$\frac{d^2y}{dx^2} = 1 - 3x^{-\frac{1}{2}}$

(iii) Find, showing all necessary working, the nature of each stationary point. [2]

Substitute x-coordinates into $\frac{d^2y}{dx^2}$

$1 - 3(4)^{-\frac{1}{2}} = -\frac{1}{2} < 0$ so $x = 4$ is maximum

$1 - 3(16)^{-\frac{1}{2}} = \frac{1}{4} > 0$ so $x = 16$ is minimum

9 A curve has equation $y = \frac{1}{x} + c$ and a line has equation $y = cx - 3$, where c is a constant.

$b^2 - 4ac = 0$

(i) Find the set of values of c for which the curve and the line meet. / one root [4]

$\frac{1}{x} + c = cx - 3$

$1 + cx = cx^2 - 3x$

$cx^2 - cx - 3x - 1 = 0$

$b^2 - 4ac = 0$

$(c+3)^2 - 4(c)(-1) = 0$

$c^2 + 6c + 9 + 4c = 0$

$c^2 + 10c + 9 = 0$

$c = -1, -9$

$c \leq -9, \ c \geqslant -1$

This means that at 2 co-ordinates, gradient of curve = gradient of line

(ii) The line is a tangent to the curve for two particular values of c. For each of these values find the x-coordinate of the point at which the tangent touches the curve. [4]

We've to find those 2 co-ordinates where such event happens.

Gradient of curve is its $\frac{dy}{dx}(-x^{-2})$

Gradient of the line is c so $-\frac{1}{x^2} = c$

Substitute this in curve's & line's equation

$$\frac{1}{x} - \frac{1}{x^2} = \frac{-1}{x^2}(x) - 3$$

$$\frac{x-1}{x^2} = \frac{-x-3x^2}{x^2} \longrightarrow 3x^2+2x-1=0$$

$$x = \frac{1}{3}, -1$$

10 Functions f and g are defined by

$$f(x) = \frac{8}{x-2} + 2 \quad \text{for } x > 2,$$

$$g(x) = \frac{8}{x-2} + 2 \quad \text{for } 2 < x < 4.$$

(i) (a) State the range of the function f. [1]

$f > 2$

(b) State the range of the function g. [1]

$g > 6$

(c) State the range of the function fg. [1]

$2 < fg < 4$

Range of $g(x)$ = domain of $f(g)$
$g(x)$'s domain covers all values that are in domains of $f(x)$

(ii) Explain why the function gf cannot be formed. [1]

Range of $f(x)$ = domain of gf but it should also ~~cover~~ cover values that are in domain of $g(x)$

Range of $f(x)$ has values 5, 6, 7, ... but domain of $g(x)$ has values less than 4.

(iii) Find the set of values of x satisfying the inequality $6f'(x) + 2f^{-1}(x) - 5 < 0$. [6]

$f(x) = \frac{8}{x-2} + 2$ so $f^{-1}(x) = \frac{8}{x-2} + 2$

$8(x-2)^{-1}$

$f'(x) = \frac{dy}{dx} = \frac{-8}{(x-2)^2}$

$6\left(\frac{-8}{(x-2)^2}\right) + 2\left(\frac{8}{x-2} + 2\right) - 5 < 0$

$\frac{-48}{(x-2)^2} + \frac{16}{x-2} + 4 - 5 < 0$

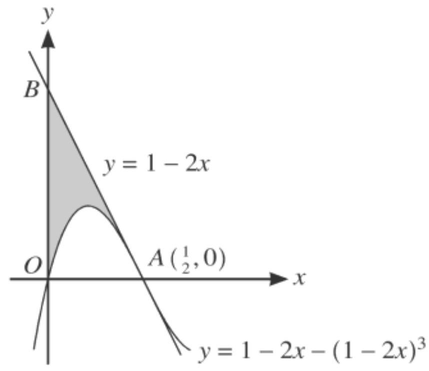
$\frac{-48}{(x-2)^2} + \frac{16}{x-2} - 1 < 0$

$x^2 - 20x + 84 < 0$

$x = 14, 6$

$x < 6, x > 14$

11



The diagram shows part of the curve $y = 1 - 2x - (1 - 2x)^3$ intersecting the x-axis at the origin O and at $A\left(\frac{1}{2}, 0\right)$. The line AB intersects the y-axis at B and has equation $y = 1 - 2x$.

(i) Show that AB is the tangent to the curve at A. [4]

This means we've to prove that gradient of AB is equal to gradient of curve at A

Gradient of curve at A $\longrightarrow \frac{dy}{dx}$ & substitute $\frac{1}{2}$

$\frac{dy}{dx} = -2 - 3(1-2x)^2 \times -2$

$= -2 + 6(1-2x)^2$ at $x = \frac{1}{2}$, $\frac{dy}{dx} = -2$

gradient of $AB = -2$

(ii) Show that the area of the shaded region can be expressed as $\int_0^{\frac{1}{2}} (1-2x)^3 \, dx$. [2]

$\int_0^{\frac{1}{2}} 1-2x \; - \int_0^{\frac{1}{2}} 1-2x-(1-2x)^3$

area under triangle OAB

area under the curve

$\left[x - x^2\right] - \left[x - x^2 - \int (1-2x)^3\right] = \int (1-2x)^3$

(iii) Hence, showing all necessary working, find the area of the shaded region. [3]

$\int_0^{\frac{1}{2}} (1-2x)^3 \longrightarrow \left[\frac{(1-2x)^4}{4} \times \frac{1}{-2}\right]_0^{\frac{1}{2}}$

$\frac{(1-2(0.5))^4}{-8} - \frac{(1-2(0))^4}{-8} = \frac{1}{8}$

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.