Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2851798509

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

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[Turn over

$S_9 = 117$

1 The sum of the first nine terms of an arithmetic progression is 117. The sum of the next four terms is 91.

Find the first term and the common difference of the progression. [4]

$S_9 = 117 \quad S_{9+4} = 117 + 91 \longrightarrow S_{13} = 208 \quad a_1 = ? \quad d = ?$

$S_n = \frac{n}{2}(2a_1 + d(n-1)) \qquad 117 = \frac{9}{2}(2a_1 + d(8)) \longrightarrow 117 = \frac{9}{2}(2a_1 + 8d)$

$208 = \frac{13}{2}(2a_1 + d(12)) \longrightarrow 208 = \frac{13}{2}(2a_1 + 12d)$

Lets make $2a_1$ the subject of the formulae

$\frac{2 \times 117}{9} - 8d_1 = \frac{208 \times 2}{13} - 12d$

$26 - 8d = 32 - 12d \longrightarrow -6 = -4d \longrightarrow d = \frac{3}{2}$

Using d, we can calculate a_1

$2a_1 = \frac{2 \times 208}{13} - 12(3/2) \longrightarrow a_1 = 7$

2 The coefficient of $\frac{1}{x}$ in the expansion of $\left(kx + \frac{1}{x}\right)^5 + \left(1 - \frac{2}{x}\right)^8$ is 74.

coefficient of x^{-1} is 74

possible to find coefficient of $\frac{\cancel{x^2}}{\cancel{x^3}} = \frac{1}{x}$

Find the value of the positive constant k. [5]

$\rightarrow 5C0 \times (kx)^5 + \left(\frac{1}{x}\right)^0 + 5C1 \times (kx)^4 \times \left(\frac{1}{x}\right)^1 + 5C2 \times (kx)^3 \times \left(\frac{1}{x}\right)^2 + 5C3 \times (kx)^2 \times \left(\frac{1}{x}\right)^3 + \ldots$

possible to find coefficient of $\frac{1}{x}$

$\rightarrow 8C0 \times (1)^8 \times \left(-\frac{2}{x}\right)^0 + \underset{4}{8C1} \times (1)^7 \times \left(-\frac{2}{x}\right)^1 + \ldots$

$5C3 \times k^2 \cancel{x^2} \times \frac{1}{\cancel{x^3}} + \left(8C1 \times -\frac{2}{x}\right) \longrightarrow \frac{10k^2}{x} - \frac{16}{x} \longrightarrow \frac{1}{x}(10k^2 - 16)$

coefficient of $\frac{1}{x} = 74 \longrightarrow 10k^2 - 16 = 74 \longrightarrow k = 3$ or -3

but since the Q asks for +ve value,

$k = 3$

3 Each year the selling price of a diamond necklace increases by 5% of the price the year before. The selling price of the necklace in the year 2000 was $36 000.

(a) Write down an expression for the selling price of the necklace n years later and hence find the selling price in 2008. [3]

$r = 5\%$ Principal amount $= \$36000$

$CI = P\left(1+\frac{R}{100}\right)^t \rightarrow 36000\left(1+\frac{5}{100}\right)^t$

$\rightarrow 36000(1.05)^t$

2000 to 2008 $\longrightarrow t = 8$

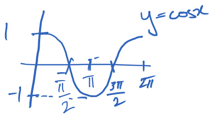
$36000(1.05)^8 = \$53188.4$

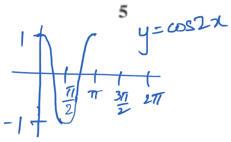
(b) The company that makes the necklace only sells one each year. Find the total amount of money obtained in the ten-year period starting in the year 2000. [2]

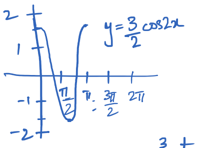
total sum $= P\left(\frac{r^t - 1}{r-1}\right) = 36000\left(\frac{1.05^{10}-1}{1.05-1}\right)$

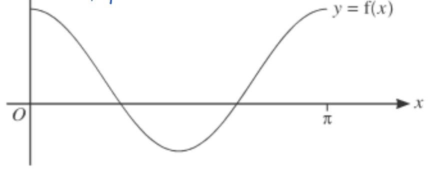
$= \$452804$

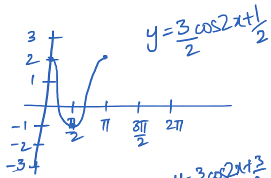
4

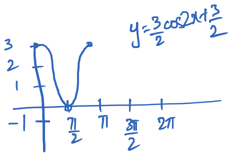












The diagram shows the graph of $y = f(x)$, where $f(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

(a) State the range of f. [2]

$-1 \leq f(x) \leq 2$

A function g is such that $g(x) = f(x) + k$, where k is a positive constant. The x-axis is a tangent to the curve $y = g(x)$.

g(x) is 1 unit up along y-axis. No transformation is along x-axis bc2 k wasn't inside the bracket f(x).

(b) State the value of k and hence describe fully the transformation that maps the curve $y = f(x)$ on to $y = g(x)$. [2]

$g(x) = \frac{3}{2}\cos 2x + \frac{1}{2} + k$ ← transformation along y-axis

X-axis can only become tangent to g(x) when f(x)'s minimum point i.e $(\frac{\pi}{2}, -1)$ shifts 1 unit up. Hence $k = 1$

$g(x) = \frac{3}{2}\cos 2x + \frac{3}{2}$. This kind of transformation is called translation by 1 unit along y-axis

(c) State the equation of the curve which is the reflection of $y = f(x)$ in the x-axis. Give your answer in the form $y = a\cos 2x + b$, where a and b are constants. [1]

reflection in x-axis → $f(x) = -g(x)$

$f(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$

Thus reflected curve → $-\frac{3}{2}\cos 2x - \frac{1}{2}$

5 The equation of a line is $y = mx + c$, where m and c are constants, and the equation of a curve is $xy = 16$.

(a) Given that the line is a tangent to the curve, express m in terms of c. [3]

$xy = 16 \rightarrow y = \frac{16}{x}$ $\quad y = mx + c$

$\frac{16}{x} = mx + c \longrightarrow mx^2 + cx - 16 = 0$

$b^2 - 4ac = 0$

$(c)^2 - 4m(-16) = 0$

$c^2 + 64m = 0$

$m = \frac{-c^2}{64}$

range for c $\quad b^2 - 4ac > 0$

(b) Given instead that $m = -4$, find the set of values of c for which the line intersects the curve at two distinct points. [3]

$y = -4x + c$ & $xy = 16 \rightarrow y = \frac{16}{x}$

$-4x + c = \frac{16}{x} \longrightarrow 4x^2 - cx + 16 = 0$

$b^2 - 4ac > 0$

$(-c)^2 - 4(4)(16) > 0$

$c^2 - 256 > 0 \longrightarrow c^2 > 256 \longrightarrow c = 16, -16$

$c > 16$ and $c < -16$

If you'd like to verify your range, substitute any value greater than 16 or less than -16, $b^2 - 4ac$ should be greater than zero.

6 Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto \tfrac{1}{2}x - a,$$
$$g : x \mapsto 3x + b,$$

where a and b are constants.

(a) Given that $gg(2) = 10$ and $f^{-1}(2) = 14$, find the values of a and b. [4]

$g(2) = 3(2) + b = 6 + b \longrightarrow g(2) = 6 + b$

$gg(2) = g(6+b) = 3(6+b) + b = 18 + 4b$

$gg(2) \longrightarrow 18 + 4b = 10 \longrightarrow b = -2$

$f(x) = \frac{1}{2}x - a$, $f^{-1}(2) = 14$ or $f(14) = 2$

x of $f^{-1}(x)$ is y of $f(x)$
and y of $f^{-1}(x)$ is x of $f(x)$

$2 = \frac{1}{2}(14) - a$

$a = 5$

(b) Using these values of a and b, find an expression for $gf(x)$ in the form $cx + d$, where c and d are constants. [2]

$g(x) = 3x - 2$, $f(x) = \frac{1}{2}x - 5$

$g\left(\frac{1}{2}x - 5\right) \longrightarrow 3\left(\frac{1}{2}x - 5\right) - 2$

$gf(x) = \frac{3}{2}x - 17$

7 (a) Prove the identity $\frac{1 + \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta} \equiv \frac{2}{\cos\theta}$. [3]

lets take LCM

$\sin^2\theta + \cos^2\theta = 1$

$$\frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$\frac{2+2\sin\theta}{\cos(1+\sin\theta)} = \frac{2\cancel{(1+\sin\theta)}}{\cos\theta\cancel{(1+\sin\theta)}} = \frac{2}{\cos\theta}$$

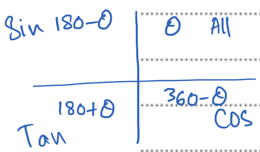
(b) Hence solve the equation $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = \frac{3}{\sin\theta}$, for $0 \leqslant \theta \leqslant 2\pi$. [3]

$\frac{2}{\cos\theta} = \frac{3}{\sin\theta}$

$2\sin\theta = 3\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{3}{2} \longrightarrow \frac{\sin\theta}{\cos\theta} = \tan\theta$

$\tan\theta = \frac{3}{2}$



$\theta = \tan^{-1}\left(\frac{3}{2}\right) \longrightarrow \theta = 0{\cdot}9828$ but since the range from 0 to 2π exists, there are going to be two answers

Since $\tan\theta$ is $+\frac{3}{2}$ not negative, we'll also take $180+\theta$ which is $\pi + 0{\cdot}9828$

$\theta = 0{\cdot}9828,\ 4{\cdot}1244$

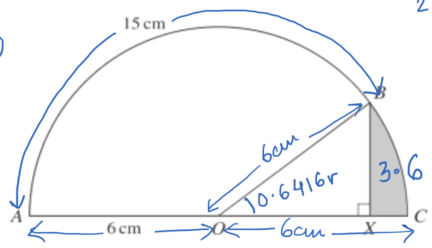
$15 = 6\theta$

$\theta = \frac{5}{2}$ radians

8

A of sector $= \frac{1}{2}r^2\theta$

$r\theta$ = arc length



OX = 4.807cm

In the diagram, ABC is a semicircle with diameter AC, centre O and radius 6 cm. The length of the arc AB is 15 cm. The point X lies on AC and BX is perpendicular to AX.

Find the perimeter of the shaded region BXC. [6]

Circumference of semicircle $= \frac{\cancel{2}\pi r}{\cancel{2}} = 6\pi$

Arc of bigger sector has a length of 15cm. This means arc of smaller sector is $6\pi - 15 = 3.85$cm$=BC$

Now that we have arc length and radius of smaller sector, we can use the formula $r\theta$ = arc length to calculate θ

$6\theta = 6\pi - 15 \longrightarrow \theta = 0.6416$ radians

By using pythagorous theorem, calculate the lengths of OX and BX

$CX = OC - OX \longrightarrow CX = 1.193$cm

Perimeter of shaded region $= CX + BX + BC$

9 The equation of a curve is $y = (3 - 2x)^3 + 24x$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

First you must differentiate the bracket & then whats inside it

$$\frac{dy}{dx} = 3(3-2x)^{3-1} \times (-2) + 24$$

$$\frac{dy}{dx} = -6(3-2x)^2 + 24$$

$$\frac{d^2y}{dx^2} = -6 \times 2(3-2x)^{2-1} \times (-2) + 0$$

$$\frac{d^2y}{dx^2} = 24(3-2x)$$

(b) Find the coordinates of each of the stationary points on the curve. [3]

For this, you should write $\frac{dy}{dx} = 0$

$-6(3-2x)^2 + 24 = 0$

$(3-2x)^2 = 4$

$3-2x = \pm 2$

$x = \frac{5}{2}$ or $x = \frac{1}{2}$

$y = 52$ or $y = 20$

$\therefore \left(\frac{5}{2}, 52\right)$ & $\left(\frac{1}{2}, 20\right)$

(c) Determine the nature of each stationary point. [2]

To do this, we must use $\frac{d^2y}{dx^2}$

If after substituting your x value in $\frac{d^2y}{dx^2}$, you get negative answer that means the co-ordinate is maximum. Same is the case for the vice versa.

$24(3-2x)$

when $x = \frac{5}{2}$, we get -48 which means $\left(\frac{5}{2}, 52\right)$ is a max. point

when $x = \frac{1}{2}$, we get 48 which means $\left(\frac{1}{2}, 20\right)$ is a minimum point.

10 The coordinates of the points A and B are $(-1, -2)$ and $(7, 4)$ respectively.

A B

(a) Find the equation of the circle, C, for which AB is a diameter. [4]

$(x-h)^2+(y-k)^2=r^2$

Centrepoint $= \frac{-1+7}{2}, \frac{-2+4}{2} = (3, 1)$

To calculate radius $\longrightarrow \sqrt{(4-1)^2+(7-3)^2}$

$= 5$

$\therefore (x-3)^2+(y-1)^2=25$

$y-1=\sqrt{25-(x-3)^2}$

$y=\sqrt{25-(x-3)^2}+1$

$y=\sqrt{25-(x^2-6x+9)}+1$

$y=\sqrt{-x^2+6x+16}+1$

$y=(-x^2+6x+16)^{\frac{1}{2}}+1$

$\frac{dy}{dx}=\frac{1}{2}(-x^2+6x+16)^{-\frac{1}{2}}\times(-2x+6)$

(b) Find the equation of the tangent, T, to circle C at the point B. [4]

$y = mx + c \longrightarrow$ equation of T

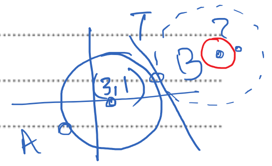
T is tangent to circle C at B means
gradient of T = gradient of circle C at B

How to calculate gradient of circle C at B ?
To calculate gradient of circle, use $\frac{dy}{dx}$ of circle's equation

Substitute x-co-ordinate of B in $\frac{dy}{dx}$ to get the gradient at B
After substitution, you'll get $-\frac{4}{3}$. T is tangent at B means
it crosses (7, 4). Now you can easily calculate
the equation $\longrightarrow 4 = -\frac{4}{3}(7) + c \longrightarrow c = \frac{40}{3} \longrightarrow 3y + 4x = 40$

(c) Find the equation of the circle which is the reflection of circle C in the line T. [3]

$(x-h)^2 + (y-k)^2 = r^2$



radius will remain
the same but midpoint
will change

We know that midpoint of circle C is (3, 1)
and B is the midpoint of co-ordinates
(3, 1) and unknown co-ordinate
Suppose unknown co-ordinate $\longrightarrow x, y$

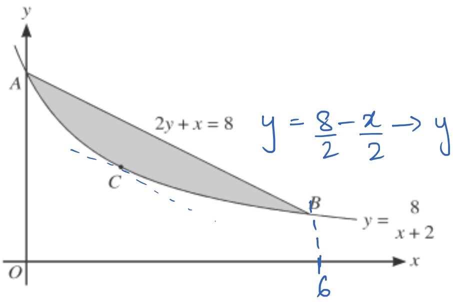
$\frac{3-x}{2} = 7 \longrightarrow x = -11$

$\frac{1-y}{2} = 4 \longrightarrow y = -7$

(−11, −7)

$(x-11)^2 + (y-7)^2 = 25$

11



$y = \frac{8}{2} - \frac{x}{2} \rightarrow y = -\frac{x}{2} + 4 \quad m = -\frac{1}{2}$

The diagram shows part of the curve $y = \frac{8}{x+2}$ and the line $2y + x = 8$, intersecting at points A and B.

The point C lies on the curve and the tangent to the curve at C is parallel to AB. ← gradient of tangent at C = gradient of AB. You also know that gradient of tangent at any point is gradient of curve at that point

(a) Find, by calculation, the coordinates of A, B and C. [6]

For A and B, we can equate equations

$\frac{8-x}{2} = \frac{8}{x+2} \longrightarrow (8-x)(x+2) = 16$

$8x + 16 - x^2 - 2x = 16$

$x^2 - 6x = 0 \longrightarrow x = 6, 0$

$y = 1, 4$

$A(0,4)$ & $B(6,1)$ //

Now to get co-ordinates of C, we know gradient of curve at C and $\frac{dy}{dx}$ of curve can be found and this way co-ordinates of C can be calculated

$y = 8(x+2)^{-1} \longrightarrow \frac{dy}{dx} = -8(x+2)^{-2}$

gradient of curve at C $= \frac{dy}{dx}$ of curve using x value of C

$-\frac{1}{2} = \frac{-8}{(x+2)^2} \longrightarrow x = 2, y = 2$ so $C(2,2)$ //

(b) Find the volume generated when the shaded region, bounded by the curve and the line, is rotated through 360° about the x-axis. [6]

For volume, we should integrate

Volume under line $\longrightarrow \pi\int_0^6 \left(-\frac{1}{2}x+4\right)^2$

$\pi\int_0^6 \frac{x^2}{4}-4x+16$

$\pi\left[\frac{x^3}{12}-\frac{4x^2}{2}+16x\right]_0^6$

Lets substitute the limits

$\pi\left[\frac{6^3}{12}-2(6)^2+16(6)\right]=42\pi$

Volume under curve $\longrightarrow \pi\int_0^6\left(\frac{8}{x+2}\right)^2$

$\pi\int_0^6 \frac{64}{x^2+4x+4}$

$\pi \times 64\int_0^6 \frac{1}{(x+2)^2}$

$64\pi\int_0^6 (x+2)^{-2}$

$64\pi\left[\frac{(x+2)^{-1}}{-1}\right]_0^6 \longrightarrow \left[\frac{-64}{x+2}\right]_0^6\pi$

After substitution, you'll get 24π

Area of Shaded region $=42\pi-24\pi=18\pi$ //

Additional Page

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