Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

JC19 06_9709_13/RP

Cambridge Assessment International Education

1 The function f is defined by $f(x) = x^2 - 4x + 8$ for $x \in \mathbb{R}$.

(i) Express $x^2 - 4x + 8$ in the form $(x - a)^2 + b$. [2]

$$\left(x - \frac{4}{2}\right)^2 - \frac{1}{4} \times (4)^2 + 8$$

$$(x-2)^2 + 4$$

(ii) Hence find the set of values of x for which $f(x) < 9$, giving your answer in exact form. [3]

$$(x-2)^2 + 4 < 9$$

$$x - 2 < \sqrt{5}$$

$$2 - \sqrt{5} < x < 2 + \sqrt{5}$$

2 (i) In the binomial expansion of $\left(2x - \frac{1}{2x}\right)^5$, the first three terms are $32x^5 - 40x^3 + 20x$. Find the remaining three terms of the expansion. [3]

$5C3 \times (2x)^2 \times \left(\frac{-1}{2x}\right)^3 + 5C4 \times (2x) \times \left(\frac{-1}{2x}\right)^4 + 5C5 \times \left(\frac{-1}{2x}\right)^5$

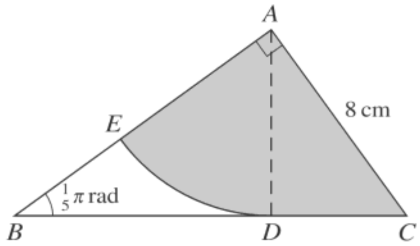
$-\frac{5}{x} + \frac{5}{8x^3} - \frac{1}{32x^5}$

(ii) Hence find the coefficient of x in the expansion of $(1 + 4x^2)\left(2x - \frac{1}{2x}\right)^5$. [2]

$(1+4x^2)\left(32x^5 - 40x^3 + 20x - 5x^{-1} + \frac{5}{8}x^{-3} - \frac{1}{32}x^{-5}\right)$

$20x - 20x = 0$

3



AD = 6.472cm
CD = 4.70247cm

The diagram shows triangle ABC which is right-angled at A. Angle $ABC = \frac{1}{5}\pi$ radians and $AC = 8$ cm. The points D and E lie on BC and BA respectively. The sector ADE is part of a circle with centre A and is such that BDC is the tangent to the arc DE at D.

(i) Find the length of AD. [3]

$$\tan \hat{ABC} = \frac{AC}{AB}$$

$$\tan\left(\frac{\pi}{5}\right) = \frac{8}{AB}$$

$$AB = 11.011\text{ cm}$$

$$\sin \hat{ABD} = \frac{AD}{AB}$$

$$\sin\left(\frac{\pi}{5}\right) = \frac{AD}{11.011} \longrightarrow AD = 6.47\text{ cm}$$

(ii) Find the area of the shaded region. [3]

$$\text{A of sector } ADE = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2} \times (6.47)^2 \times \frac{3\pi}{10} = 19.74\text{ cm}^2$$

$$\text{A of triangle } ACD = \frac{1}{2} \times 6.472 \times 4.702 = 15.22\text{ cm}^2$$

$$\text{A of shaded region} = 19.74 + 15.22$$

$$= 34.96\text{ cm}^2$$

$$\approx 35.0\text{ cm}^2$$

4 The function f is defined by $f(x) = \dfrac{48}{x-1}$ for $3 \leqslant x \leqslant 7$. The function g is defined by $g(x) = 2x - 4$ for $a \leqslant x \leqslant b$, where a and b are constants.

(i) Find the greatest value of a and the least value of b which will permit the formation of the composite function gf. [2]

f(x) has domain of $3 \leq x \leq 7$ and range of $8 \leq y \leq 24$. f(x) is substituted in g(x) so range values of f(x) will become domain of g(x) in gf function.

$a = 8$ & $b = 24$

It is now given that the conditions for the formation of gf are satisfied.

(ii) Find an expression for $gf(x)$. [1]

$g\left(\dfrac{48}{x-1}\right) = 2\left(\dfrac{48}{x-1}\right) - 4 = \dfrac{96}{x-1} - 4$

(iii) Find an expression for $(gf)^{-1}(x)$. [2]

$gf(x) = \dfrac{96}{x-1} - 4$

$(y+4)(x-1) = 96 \longrightarrow x = \dfrac{96}{y+4} + 1$

$(gf)^{-1}(x) = \dfrac{96}{x+4} + 1$

5 Two heavyweight boxers decide that they would be more successful if they competed in a lower weight class. For each boxer this would require a total weight loss of 13 kg. At the end of week 1 they have each recorded a weight loss of 1 kg and they both find that in each of the following weeks their weight loss is slightly less than the week before.

Boxer A's weight loss in week 2 is 0.98 kg. It is given that his weekly weight loss follows an arithmetic progression.

changes by fixed difference

(i) Write down an expression for his total weight loss after x weeks. [1]

1, 0.98, 0.96, ... until x weeks $d = 0.02$

$S_n = \frac{n}{2}(2a_1 + d(n-1))$

$S_n = \frac{x}{2}(2 + 0.02(x-1))$

$S_n = 0.99x + 0.01x^2$

(ii) He reaches his 13 kg target during week n. Use your answer to part (i) to find the value of n. [2]

1, 0.98, 0.96, ...

$0.99n + 0.01n^2 = 13$

$n = 16$

Boxer B's weight loss in week 2 is 0.92 kg and it is given that his weekly weight loss follows a geometric progression. by a fixed %

(iii) Calculate his total weight loss after 20 weeks and show that he can never reach his target. [4]

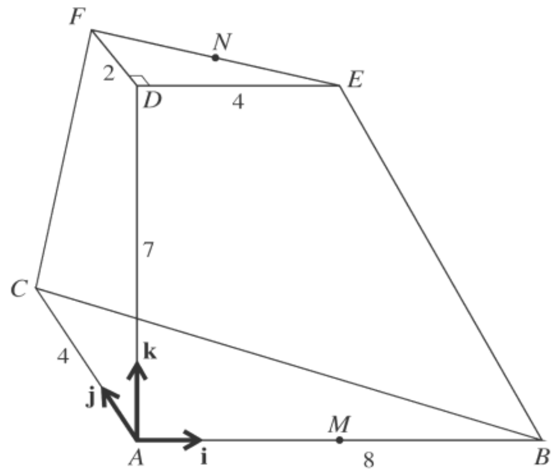
$1, 0.92, 0.8464, \dots \quad r = 0.92$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_n = \frac{1(1-0.92^{20})}{1-0.92} = 10.14\,kg$

$S_\infty = \frac{a_1}{1-r} = \frac{1}{1-0.92} = 12.5\,kg < 13\,kg$

6



The diagram shows a solid figure $ABCDEF$ in which the horizontal base ABC is a triangle right-angled at A. The lengths of AB and AC are 8 units and 4 units respectively and M is the mid-point of AB. The point D is 7 units vertically above A. Triangle DEF lies in a horizontal plane with DE, DF and FE parallel to AB, AC and CB respectively and N is the mid-point of FE. The lengths of DE and DF are 4 units and 2 units respectively. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ respectively.

(i) Find $\overrightarrow{MF}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [1]

$\overrightarrow{MF} = \overrightarrow{MA} + \overrightarrow{AD} + \overrightarrow{DF}$

$= -4i + 7k + 2j$

$\overrightarrow{MF} = -4i + 2j + 7k$

(ii) Find $\overrightarrow{FN}$ in terms of $\mathbf{i}$ and $\mathbf{j}$. [1]

$\overrightarrow{FN} = \dfrac{\overrightarrow{FE}}{2} \longrightarrow \overrightarrow{FE} = \overrightarrow{FD} + \overrightarrow{DE}$

$\overrightarrow{FE} = -2j + 4i$

$\overrightarrow{FN} = 2i - j$

(iii) Find $\overrightarrow{MN}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [1]

$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AD} + \overrightarrow{DF} + \overrightarrow{FN}$

$= -4i + 7k + 2j + 2i - j$

$\overrightarrow{MN} = -2i + j + 7k$

(iv) Use a scalar product to find angle FMN. [4]

$\overrightarrow{FM} = 4i - 2j - 7k$

$\overrightarrow{NM} = 2i - j - 7k$

$\sqrt{(4)^2 + (-2)^2 + (-7)^2} \times \sqrt{(2)^2 + (-1)^2 + (-7)^2} \cos\theta = \begin{pmatrix} 4 \\ -2 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -7 \end{pmatrix}$

$\cos\theta = \dfrac{59}{9\sqrt{46}}$

$\theta = 14.9^\circ$

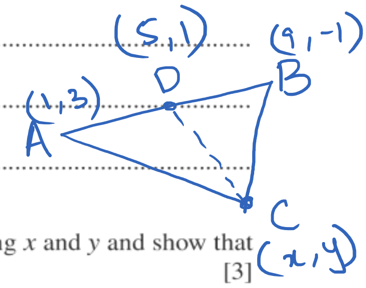
7 The coordinates of two points A and B are $(1,\ 3)$ and $(9,\ -1)$ respectively and D is the mid-point of AB. A point C has coordinates $(x,\ y)$, where x and y are variables.

(i) State the coordinates of D. [1]

$$\frac{1+9}{2},\ \frac{3+(-1)}{2} \longrightarrow D(5,1)$$

(ii) It is given that CD^2 (Length)^2 $= 20$. Write down an equation relating x and y. [1]

$$(y-1)^2 + (x-5)^2 = 20$$



(iii) It is given that AC and BC are equal in length. Find an equation relating x and y and show that it can be simplified to $y = 2x - 9$. [3]

Using same method from (ii)

length of AC = length of BC

$$(y-3)^2 + (x-1)^2 = (y+1)^2 + (x-9)^2$$

$$\cancel{y^2} - 6y + 9 + \cancel{x^2} - 2x + 1 = \cancel{y^2} + 2y + 1 + \cancel{x^2} - 18x + 81$$

$$-6y + 10 - 2x = 2y + 82 - 18x$$

$$16x = 8y + 72$$

$$\frac{16x}{8} - \frac{72}{8} = y$$

$$y = 2x - 9$$

(iv) Using the results from parts **(ii)** and **(iii)**, and showing all necessary working, find the possible coordinates of C. [4]

$5x^2 - 50x + 105 = 0$

$x = 3$ or $x = 7$

$y = -3$ or $y = 5$

8 A curve is such that $\frac{dy}{dx} = 3x^2 + ax + b$. The curve has stationary points at $(-1, 2)$ and $(3, k)$. Find the values of the constants a, b and k. [8]

$\frac{dy}{dx} = 0$ to find stationary points

① $3(-1)^2 - a + b = 0$

$b - a = -3$

② $3(3)^2 + 3a + b = 0$

$3a + b = -27$

$-27 - 3a = -3 + a$

$a = -6$, $b = -9$

$\int 3x^2 + ax + b \longrightarrow x^3 + \frac{ax^2}{2} + bx + c = y$

$y = x^3 - 3x^2 - 9x + c$ Use $(-1, 2)$

$2 = (-1)^3 - 3(-1)^2 - 9(-1) + c \longrightarrow c = -3$

$y = x^3 - 3x^2 - 9x - 3$

substitute 3 to get value of k

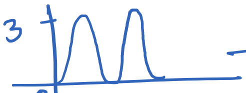
$y = (3)^3 - 3(3)^2 - 9(3) - 3$

$y = -30$

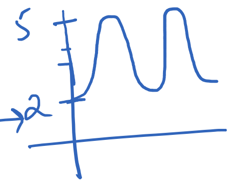
$k = -30$

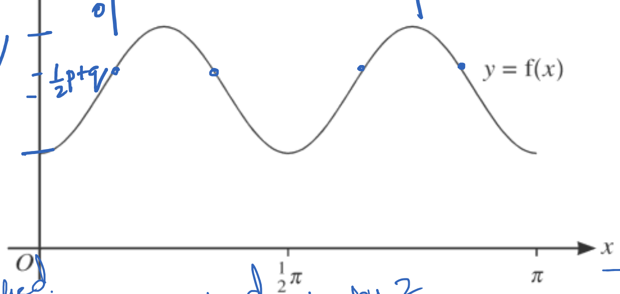
9 $p=3, q=2 \rightarrow$ Suppose $y=3\sin^2 2x+2$

$3\sin^2 2x$

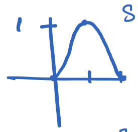


$3\sin^2 2x+2$

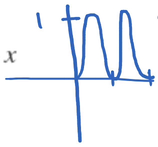




$\sin^2 x = y$



$\sin^2 2x = y$



stretched along y-axis

shrinked along x-axis by 2

The function $\mathrm{f} : x \mapsto p\sin^2 2x + q$ is defined for $0 \leqslant x \leqslant \pi$, where p and q are positive constants. The diagram shows the graph of $y = \mathrm{f}(x)$. moved upwards q units

(i) In terms of p and q, state the range of f. [2]

$q \leq f(x) \leq p+q$

$y=\sin^2 2x$ has range of $0 \leq y \leq 1$ when $\sin^2 2x$ multiplies with p, range becomes $0 \leq y \leq p$. But when $\sin^2 2x$ moves upwards by q units, range becomes $q \leq y \leq p+q$

(ii) State the number of solutions of the following equations.

(a) $\mathrm{f}(x) = p + q$ [1]

$\frac{\pi}{4}, \frac{3\pi}{4}$ so 2 solutions

(b) $\mathrm{f}(x) = q$ [1]

$0, \frac{\pi}{2}, \pi$ so 3 solutions

(c) $\mathrm{f}(x) = \frac{1}{2}p + q$ [1]

$\frac{1}{2}\cancel{p}+\cancel{q} = \cancel{p}\sin^2 2x+\cancel{q}$

$\sin 2x = \frac{1}{\sqrt{2}}$ $\quad x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$ so 4 solutions

$x = \frac{45}{2}, \frac{135}{2}$

(iii) For the case where $p = 3$ and $q = 2$, solve the equation $f(x) = 4$, showing all necessary working. [5]

$y = 3\sin^2 2x + 2$

$3\sin^2 2x = 2$

$\sin 2x = \frac{\sqrt{6}}{3}$

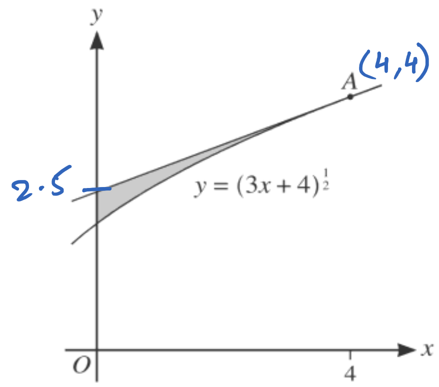
$\sin x = \frac{\sqrt{6}}{3} \longrightarrow x = 54.74, 125.26, \ldots$

For $\sin 2x$, $x = \frac{54.74}{2}$ and $\frac{125.26}{2}, \ldots$

$x = 0.478$ & 1.09

2.05 & 2.66

10



The diagram shows part of the curve with equation $y = (3x + 4)^{\frac{1}{2}}$ and the tangent to the curve at the point A. The x-coordinate of A is 4.

(i) Find the equation of the tangent to the curve at A. [5]

For equation of tangent, we need to know its gradient.

Gradient of tangent = gradient of curve at A

$$\frac{dy}{dx} = \frac{1}{2}(3x+4)^{-\frac{1}{2}} \times 3$$

$$\frac{1}{2}(3(4)+4)^{-\frac{1}{2}} \times 3 \longrightarrow m = \frac{3}{8}$$

$m = \frac{3}{8}$ & $(4,4)$

$$y - 4 = \frac{3}{8}(x-4)$$

$$y = \frac{3}{8}x + \frac{5}{2}$$

(ii) Find, showing all necessary working, the area of the shaded region. [5]

$$\int_0^4 (3x+4)^{\frac{1}{2}}$$

$$\left[\frac{2(3x+4)^{\frac{3}{2}}}{3} \times \frac{1}{3}\right]_0^4 = \left[\frac{2(3x+4)^{\frac{3}{2}}}{9}\right]_0^4$$

$$\frac{128}{9} - \frac{16}{9} = \frac{112}{9}$$

Area under tangent is area of trapezium

$$\frac{1}{2} \times 4 \times (4+2.5) = 13$$

A of shaded region $= \frac{112}{9} - 13$

$= 0.556$

[Question 10 (iii) is printed on the next page.]

(iii) A point is moving along the curve. At the point P the y-coordinate $\frac{dy}{dt}$ is increasing at half the rate at which the x-coordinate $\frac{dx}{dt}$ is increasing. Find the x-coordinate of P. [3]

At P, $\frac{dy}{dt} = \frac{dx}{2dt}$

$$y = (3x+4)^{\frac{1}{2}} \longrightarrow \frac{dy}{dx} = \frac{1}{2}(3x+4)^{-\frac{1}{2}} \times 3$$

$$\frac{dy}{dx} = \frac{\cancel{dt}}{2\cancel{dt}} \longrightarrow \frac{dy}{dx} = \frac{1}{2}$$

$$\frac{3}{2}(3x+4)^{-\frac{1}{2}} = \frac{1}{2}$$

$$\frac{1}{\sqrt{3x+4}} = \frac{1}{3} \longrightarrow 9 = 3x+4$$

$$x = 5/3$$

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

BLANK PAGE