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CANDIDATE NAME

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.

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1 Find the coefficient of x in the expansion of $\left(\frac{2}{x} - 3x\right)^5$. [3]

$5C0 \times \left(\frac{2}{x}\right)^5 + 5C1 \times \left(\frac{2}{x}\right)^4 \times (-3x) + 5C2 \times \left(\frac{2}{x}\right)^3 \times (-3x)^2 + 5C3 \times \left(\frac{2}{x}\right)^2 \times (-3x)^3$

$5C3 \times \frac{4}{\cancel{x^2}} \times -27x^{\cancel{3}} = -1080x$

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2 Two points A and B have coordinates $(1, 3)$ and $(9, -1)$ respectively. The perpendicular bisector of AB intersects the y-axis at the point C. Find the coordinates of C. [5]

$A(1,3) \quad B(9,-1)$

m of $AB \longrightarrow \frac{-1-3}{9-1} = -\frac{1}{2}$

perpendicular bisector of AB has gradient of 2 and crosses centre of AB i.e $(5,1)$

Equation of perp. bisector of AB $\longrightarrow y-1 = 2(x-5)$

$y = 2x - 9$

On y-axis, $x = 0$ $\quad \frac{y+9}{2} = 0 \longrightarrow y = -9$, $x = 0$

$C(0,-9)$

3 A curve is such that $\frac{dy}{dx} = x^3 - \frac{4}{x^2}$. The point $P\,(2, 9)$ lies on the curve.

$\frac{dx}{dt} = -0.05$

(i) A point moves on the curve in such a way that the x-coordinate is decreasing at a constant rate of 0.05 units per second. Find the rate of change of the y-coordinate when the point is at P. [2]

$\frac{dy}{dt} = ?$

$\frac{dy}{dx} = x^3 - \frac{4}{x^2}$ Substitute $(2,9)$ in $\frac{dy}{dx}$

$\frac{dy}{dx} = 7$ $\quad \cancel{\frac{dy}{dx}} \times \cancel{\frac{dx}{dt}} = \frac{dy}{dt}$

$7 \times -0.05 = \frac{dy}{dt}$

$\frac{dy}{dt} = -0.35$ //

(ii) Find the equation of the curve. [3]

$\frac{dy}{dx} = x^3 - 4x^{-2}$

$\int x^3 - 4x^{-2} \longrightarrow \frac{x^4}{4} - \frac{4x^{-1}}{-1} + c = \frac{x^4}{4} + \frac{4}{x} + c$

$y = \frac{x^4}{4} + \frac{4}{x} + c$ Use $(2,9)$

$9 = \frac{(2)^4}{4} + \frac{4}{2} + c \longrightarrow c = 3$

$y = \frac{x^4}{4} + \frac{4}{x} + 3$

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4 Angle x is such that $\sin x = a + b$ and $\cos x = a - b$, where a and b are constants.

(i) Show that $a^2 + b^2$ has a constant value for all values of x. [3]

$\sin^2 x = (a+b)^2$ $\quad\quad$ $\cos^2 x = (a-b)^2$

$\sin^2 x = a^2+2ab+b^2$ $\quad\quad$ $\cos^2 x = a^2-2ab+b^2$

$\sin^2 x - 2ab = a^2+b^2$ $\quad\quad$ $\cos^2 x + 2ab = a^2+b^2$

$2a^2+2b^2 = \sin^2 x + \cos^2 x + \cancel{2ab} - \cancel{2ab}$

$2(a^2+b^2) = 1$

$a^2+b^2 = \frac{1}{2}$

(ii) In the case where $\tan x = 2$, express a in terms of b. [2]

$\frac{\sin x}{\cos x} = 2$

$\frac{a+b}{a-b} = 2 \longrightarrow a+b = 2a-2b$

$a = 3b$

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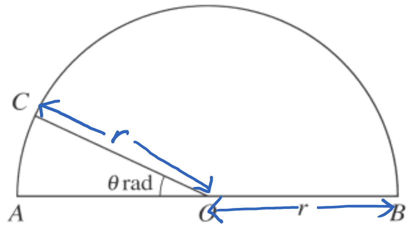
5



The diagram shows a semicircle with diameter AB, centre O and radius r. The point C lies on the circumference and angle $AOC = \theta$ radians. The perimeter of sector BOC is twice the perimeter of sector AOC. Find the value of θ correct to 2 significant figures. [5]

Arc length = $r\theta$

Perimeter of sector BOC = $r(\pi - \theta) + 2r$

Perimeter of sector AOC = $r\theta + 2r$

$$r(\pi - \theta) + 2r = 2(r\theta + 2r)$$

$$\pi r - r\theta + 2r = 2r\theta + 4r$$

$$\cancel{r}(\pi - \theta + 2) = \cancel{r}(2\theta + 4)$$

$$\pi + 2 - \theta = 2\theta + 4$$

$$\pi - 2 = 3\theta$$

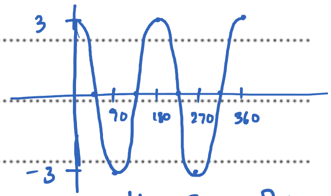
$$\theta = \frac{\pi - 2}{3} = 0.38 \text{ radians}$$

6 The equation of a curve is $y = 3\cos 2x$ and the equation of a line is $2y + \dfrac{3x}{\pi} = 5$.

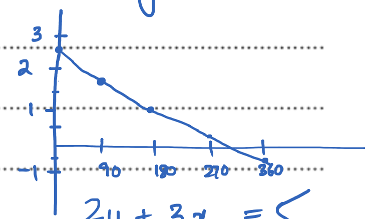
(i) State the smallest and largest values of y for both the curve and the line for $0 \leqslant x \leqslant 2\pi$. [3]

for $y = 3\cos 2x$, $-3 \leq y \leq 3$

for $2y + \dfrac{3x}{\pi} = 5$, $-\dfrac{1}{2} \leq y \leq \dfrac{5}{2}$

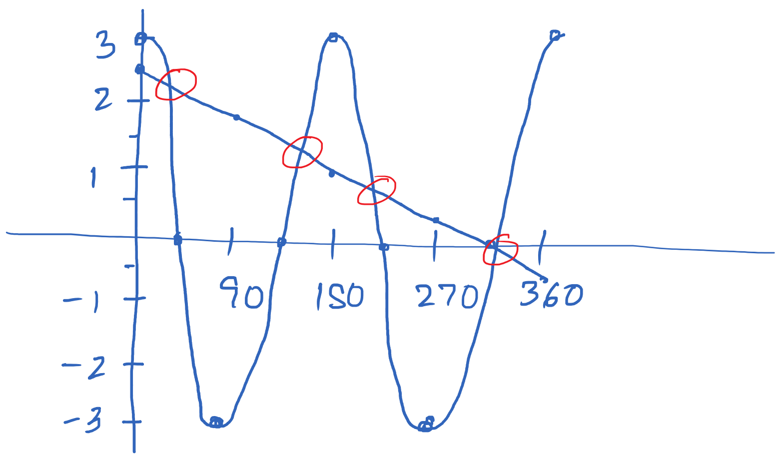


$y = 3\cos 2x$



$2y + \dfrac{3x}{\pi} = 5$

(ii) Sketch, on the same diagram, the graphs of $y = 3\cos 2x$ and $2y + \dfrac{3x}{\pi} = 5$ for $0 \leqslant x \leqslant 2\pi$. [3]



(iii) State the number of solutions of the equation $6\cos 2x = 5 - \dfrac{3x}{\pi}$ for $0 \leqslant x \leqslant 2\pi$. [1]

$6\cos 2x = 5 - \dfrac{3x}{\pi}$ ← This basically is $3\cos 2x = \dfrac{\left(5 - \dfrac{3x}{\pi}\right)}{2}$

meaning how many times both graphs intersect

4 solutions

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7 Functions f and g are defined by

$$f : x \mapsto 3x - 2, \quad x \in \mathbb{R},$$

$$g : x \mapsto \frac{2x+3}{x-1}, \quad x \in \mathbb{R},\ x \neq 1.$$

(i) Obtain expressions for $f^{-1}(x)$ and $g^{-1}(x)$, stating the value of x for which $g^{-1}(x)$ is not defined. [4]

$y = 3x - 2 \longrightarrow \frac{y+2}{3} = x$

$f^{-1}(x) = \frac{x+2}{3}$

$y = \frac{2x+3}{x-1} \longrightarrow x = \frac{3+y}{y-2}$

$g^{-1}(x) = \frac{3+x}{x-2} \quad x \neq 2$

bcz if $x=2$ then denominator becomes zero, it which makes invalid

(ii) Solve the equation $\mathrm{fg}(x) = \frac{7}{3}$. [3]

$$f\left(\frac{2x+3}{x-1}\right) = \frac{7}{3}$$

$$3\left(\frac{2x+3}{x-1}\right) - 2 = \frac{7}{3}$$

$$\frac{2x+3}{x-1} = \frac{13}{9}$$

$$18x + 27 = 13x - 13$$

$$5x = -40$$

$$x = -8$$

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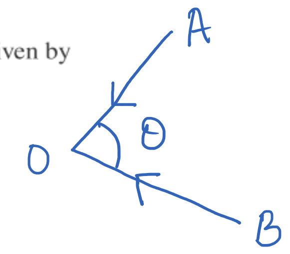
8 The position vectors of points A and B, relative to an origin O, are given by

$$\overrightarrow{OA} = \begin{pmatrix} 6 \\ -2 \\ -6 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ k \\ -3 \end{pmatrix},$$



where k is a constant.

(i) Find the value of k for which angle AOB is 90°. [2]

$\overrightarrow{AO} = -6i + 2j + 6k$

$\overrightarrow{BO} = -3i - kj + 3k$

$$\begin{pmatrix} -6 \\ 2 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ -k \\ 3 \end{pmatrix} = 0$$

$18 - 2k + 18 = 0$

$k = 18$

(ii) Find the values of k for which the lengths of OA and OB are equal. [2]

$$\sqrt{(6)^2 + (2)^2 + (6)^2} = \sqrt{(3)^2 + (-k)^2 + (3)^2}$$

$76 = 18 + k^2$

$k = \pm\sqrt{58}$

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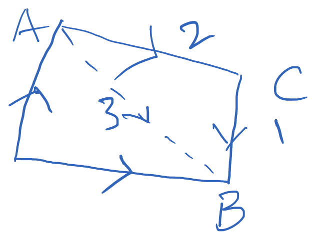
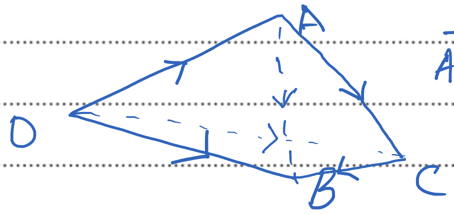
The point C is such that $\overrightarrow{AC} = 2\overrightarrow{CB}$.

$AC : CB$
$2 : 1$



(iii) In the case where $k = 4$, find the unit vector in the direction of $\overrightarrow{OC}$. [4]



$\overrightarrow{AC} = 2\overrightarrow{CB}$

$\overrightarrow{OA} = 6i - 2j - 6k \qquad \overrightarrow{OB} = 3i + 4j - 3k$

$\overrightarrow{AB} = -6i + 2j + 6k + 3i + 4j - 3k$

$\overrightarrow{AB} = -3i + 6j + 3k$

$\overrightarrow{AC} = \frac{2}{3}(-3i + 6j + 3k)$

$\overrightarrow{AC} = -2i + 4j + 2k$

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$= 4i + 2j - 4k$

$\sqrt{(4)^2 + (2)^2 + (-4)^2} = 6$

$\frac{1}{6}(4i + 2j - 4k)$

9 The curve C_1 has equation $y = x^2 - 4x + 7$. The curve C_2 has equation $y^2 = 4x + k$, where k is a constant. The tangent to C_1 at the point where $x = 3$ is also the tangent to C_2 at the point P. Find the value of k and the coordinates of P. [8]

$y=4$

gradient of C_1 at $x=3$ is equal to gradient of that tangent

gradient of C_1 $= \frac{dy}{dx}$ of C_1

$y = x^2 - 4x + 7 \longrightarrow \frac{dy}{dx} = 2x - 4$

At $x = 3$, gradient of C_1 is $2(3) - 4 = 2$

Equation of that tangent $\longrightarrow y - 4 = 2(x-3)$

$y = 2x - 2$

That same tangent is also tangent at C_2 at P meaning they intersect at P

$2x - 2 = \sqrt{4x+k}$

$4x^2 - 8x + 4 = 4x + k$

$4x^2 - 12x + 4 - k = 0$

Use $b^2 - 4ac = 0$

$(-12)^2 - 4(4)(4-k) = 0$

$144 - 64 + 16k = 0$

$k = -5$

$y^2 = 4x - 5$ & $y = 2x - 2$

$4x - 5 = 4x^2 - 8x + 4$

$4x^2 - 12x + 9 = 0$

$x = \frac{3}{2}, y = 1 \longrightarrow P\left(\frac{3}{2}, 1\right)$

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$S_{10} = S_{15} - S_{10}$

10 **(a)** In an arithmetic progression, the sum of the first ten terms is equal to the sum of the next five terms. The first term is a.

(i) Show that the common difference of the progression is $\frac{1}{3}a$. [4]

$a_1 = a \qquad S_n = \frac{n}{2}(2a_1 + d(n-1))$

$\cancel{\frac{10}{2}}^{5}(2a+9d) = \frac{15}{2}(2a+14d) - \cancel{\frac{10}{2}}^{5}(2a+9d)$

$10a + 45d = 15a + 105d - 10a - 45d$

$20a + 90d = 15a + 105d$

$5a = 15d$

$d = \frac{a}{3}$

(ii) Given that the tenth term is 36 more than the fourth term, find the value of a. [2]

$a_{10} = 36 + a_4 \qquad a_1 = ?$

$\cancel{a_1} + 9d = 36 + \cancel{a_1} + 3d$

$6d = 36$

$d = 6$

$a_1 = 18$

(b) The sum to infinity of a geometric progression is 9 times the sum of the first four terms. Given that the first term is 12, find the value of the fifth term. [4]

$S_\infty = \dfrac{a}{1-r}$ $\qquad S_n = \dfrac{a(1-r^n)}{1-r}$ $\qquad a_1 = 12$, $a_5 = ?$

$$\frac{a}{1-r} = 9\,\frac{a(1-r^n)}{1-r}$$

$$\frac{12}{1-r} = \frac{108(1-r^4)}{1-r}$$

$$12 = 108 - 108r^4$$

$$96 = 108r^4$$

$$r = \sqrt[4]{\frac{96}{108}}$$

$$a_5 = a_1 \times r^{n-1}$$

$$= 12 \times \left(\sqrt[4]{\frac{96}{108}}\right)^{5-1} \rightarrow a_5 = \frac{32}{3}$$

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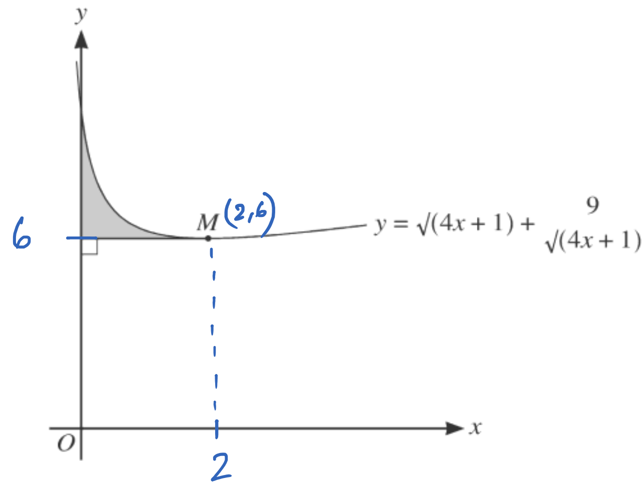

The diagram shows part of the curve $y = \sqrt{(4x+1)} + \dfrac{9}{\sqrt{(4x+1)}}$ and the minimum point M.

(i) Find expressions for $\dfrac{dy}{dx}$ and $\int y\,dx$. [6]

$$y = (4x+1)^{\frac{1}{2}} + 9(4x+1)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}}(4x+1)^{-\frac{1}{2}} \times \cancel{4}^{2} - \left(\frac{9}{\cancel{2}}(4x+1)^{-\frac{3}{2}} \times \cancel{4}^{2}\right)$$

$$= 2(4x+1)^{-\frac{1}{2}} - 18(4x+1)^{-\frac{3}{2}}$$

$$\int (4x+1)^{\frac{1}{2}} + 9\int (4x+1)^{-\frac{1}{2}}$$

$$\left[\frac{\cancel{2}^{1}}{3}(4x+1)^{\frac{3}{2}} \times \frac{1}{\cancel{4}_{2}}\right] + 9\left[\cancel{2}(4x+1)^{\frac{1}{2}} \times \frac{1}{\cancel{4}_{2}}\right]$$

$$\frac{(4x+1)^{\frac{3}{2}}}{6} + \frac{9(4x+1)^{\frac{1}{2}}}{2} + c$$

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(ii) Find the coordinates of M. [3]

$\frac{dy}{dx} = 0$

$= 2(4x+1)^{-\frac{1}{2}} - 18(4x+1)^{-\frac{3}{2}} = 0$

$\frac{2}{\sqrt{4x+1}} - \frac{18}{(4x+1)^{\frac{3}{2}}} = 0$

$\frac{2}{\sqrt{4x+1}} = \frac{18}{(4x+1)^{\frac{3}{2}}}$

$2(4x+1)^{\frac{3}{2}} = 18(4x+1)^{\frac{1}{2}}$

$(4x+1) = 9 \quad x = 2, y = 6$

The shaded region is bounded by the curve, the y-axis and the line through M parallel to the x-axis.

(iii) Find, showing all necessary working, the area of the shaded region. [3]

A of shaded region $= \int_0^2$ curve $-$ A of rectangle

$\int (4x+1)^{\frac{1}{2}} + 9\int (4x+1)^{-\frac{1}{2}}$

$\left[\frac{\cancel{2}^{1}}{3}(4x+1)^{\frac{3}{2}} \times \frac{1}{\cancel{4}_{2}}\right] + 9\left[\cancel{2}(4x+1)^{\frac{1}{2}} \times \frac{1}{\cancel{4}_{2}}\right]$

$\left[\frac{(4x+1)^{\frac{3}{2}}}{6} + \frac{9(4x+1)^{\frac{1}{2}}}{2}\right]_0^2 = 18 - \frac{14}{3} = \frac{40}{3}$

A of rectangle $= 2 \times 6 = 12$

A of shaded region $= \frac{40}{3} - 12 = \frac{4}{3}$

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Additional Page

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