Cambridge International AS & A Level

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2339823814

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **21** printed pages and **3** blank pages.

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1 The term independent of x in the expansion of $\left(2x + \frac{k}{x}\right)^6$, where k is a constant, is 540.

(i) Find the value of k. [3]

$6C0 \times (2x)^6 \times \left(\frac{k}{x}\right)^0 + 6C1 \times (2x)^5 \times \frac{k}{x} +$

$6C2 \times (2x)^4 \times \frac{k^2}{x^2} + 6C3 \times (2x)^3 \times \frac{k^3}{x^3}$

$6C3 \times 8\cancel{x^3} \times \frac{k^3}{\cancel{x^3}} = 160k^3$

$160k^3 = 540$

$k = \frac{3}{2}$

(ii) For this value of k, find the coefficient of x^2 in the expansion. [2]

$\left(2x + \frac{1.5}{x}\right)^6$

$6C0 \times (2x)^6 + 6C1 \times (2x)^5 \times \frac{1.5}{x} + 6C2 \times (2x)^4 \times \frac{(1.5)^2}{x^2}$

$6C2 \times 16\cancel{x^4} \times \frac{(1.5)^2}{\cancel{x^2}} = 540x^2$

2 The line $4y = x + c$, where c is a constant, is a tangent to the curve $y^2 = x + 3$ at the point P on the curve.

(i) Find the value of c. [3]

$$\frac{x+c}{4} = \sqrt{x+3}$$

$$x^2 + 2cx + c^2 = 16x + 48$$

$$x^2 + 2cx - 16x + c^2 - 48 = 0$$

$$b^2 - 4ac = 0$$

$$a = 1,\ b = 2c - 16,\ c = c^2 - 48$$

$$(2c-16)^2 - 4(c^2 - 48) = 0$$

$$\cancel{4c^2} - 64c + 256 - \cancel{4c^2} + 192 = 0$$

$$-64c = -448$$

$$c = 7$$

(ii) Find the coordinates of P. [2]

$$\frac{x+7}{4} = \sqrt{x+3}$$

$$x^2 + 14x + 49 = 16x + 48$$

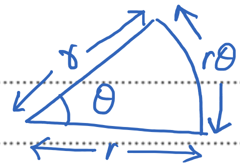
$$x^2 - 2x + 1 = 0$$

$$x = 1,\ y = 2$$

$$P(1, 2)$$

3 A sector of a circle of radius r cm has an area of A cm^2. Express the perimeter of the sector in terms of r and A. [4]

$\frac{1}{2} r^2 \theta = A$



Arc length $= r\theta$

$\frac{r^2\theta}{2} = A$ Make θ the subject of formula

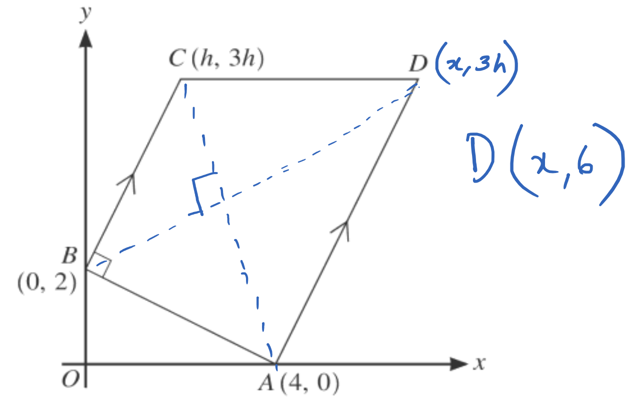
$\frac{2A}{r^2} = \theta$

Perimeter $= r + r + r\theta$

$= 2r + r\left(\frac{2A}{r^2}\right)$

$= 2r + \frac{2A}{r}$

4



The diagram shows a trapezium $ABCD$ in which the coordinates of A, B and C are (4, 0), (0, 2) and $(h, 3h)$ respectively. The lines BC and AD are parallel, angle $ABC = 90°$ and CD is parallel to the x-axis.

(i) Find, by calculation, the value of h. [3]

gradient of AB x gradient of BC = −1

$\frac{0-2}{4-0} \times \frac{2-3h}{0-h} = -1$

$2 - 3h = -2h$

$2 = h$

(ii) Hence find the coordinates of D. [3]

gradient of BC = gradient of AD

B(0,2) C(2,6) m=2 $\frac{6-0}{x-4}=2$

$x=7$, $y=6$

D(7,6)

5 The function f is defined by $f(x) = -2x^2 + 12x - 3$ for $x \in \mathbb{R}$.

(i) Express $-2x^2 + 12x - 3$ in the form $-2(x+a)^2 + b$, where a and b are constants. [2]

Make coefficient of x^2 1 & keep -3 separate

$-2(x^2 - 6x) - 3$

$-2\left[\left(x - \frac{6}{2}\right)^2 - \frac{1}{4} \times (-6)^2\right] - 3$

$-2(x-3)^2 + 15$

(ii) State the greatest value of $f(x)$. [1]

This comes from answer of previous part

$-2(x-3)^2 + 15$

$f(x) \leq 15$

The function g is defined by $g(x) = 2x + 5$ for $x \in \mathbb{R}$.

(iii) Find the values of x for which $gf(x) + 1 = 0$. [3]

$g(-2x^2+12x-3)+1=0$

$2(-2x^2+12x-3)+5+1=0$

$-4x^2+24x\cancel{-6}\cancel{+6}=0$

$4x=24$

$x=6$ or 0

6 (i) Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)^2 \equiv \frac{1 - \sin x}{1 + \sin x}$. [4]

$$\frac{1}{\cos^2 x} - \frac{2\tan x}{\cos x} + \tan^2 x$$

$$\frac{1}{\cos^2 x} - \frac{2\sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$$

$$\frac{1 - 2\sin x + \sin^2 x}{\cos^2 x}$$

$$\cos^2 x = (1+\sin x)(1-\sin x)$$

$$\frac{\sin^2 x - 2\sin x + 1}{(1+\sin x)(1-\sin x)}$$

$$\frac{(1-\sin x)\cancel{(1-\sin x)}}{(1+\sin x)\cancel{(1-\sin x)}} = \frac{1-\sin x}{1+\sin x}$$

(ii) Hence solve the equation $\left(\frac{1}{\cos 2x} - \tan 2x\right)^2 = \frac{1}{3}$ for $0 \leqslant x \leqslant \pi$. [3]

$$\frac{1 - \sin 2x}{1 + \sin 2x} = \frac{1}{3}$$

$$3 - 3\sin 2x = 1 + \sin 2x$$

$$2 = 4\sin 2x$$

$$\sin 2x = \frac{1}{2}$$

Lets assume $\sin x = \frac{1}{2}$

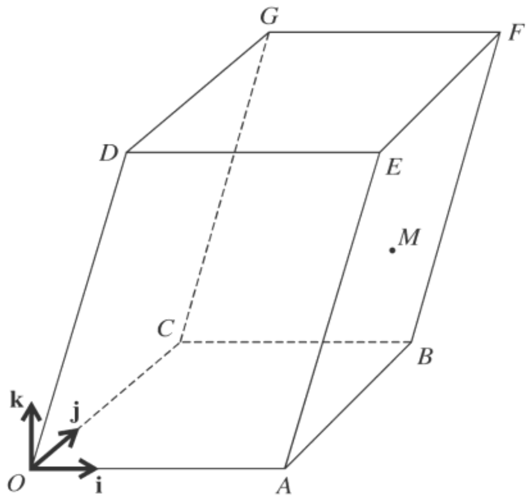
$$x = \pi/6 \ , \ \frac{5}{6}\pi$$

$$\sin x = \frac{\pi}{6} \ , \ \frac{5\pi}{6}$$

$$\sin 2x = \left(\frac{\pi}{6}\right) \div 2 \ , \ \left(\frac{5\pi}{6}\right) \div 2$$

$$\sin 2x = \frac{\pi}{12} \ , \ \frac{5\pi}{12}$$

7



The diagram shows a three-dimensional shape in which the base $OABC$ and the upper surface $DEFG$ are identical horizontal squares. The parallelograms $OAED$ and $CBFG$ both lie in vertical planes. The point M is the mid-point of AF.

Unit vectors **i** and **j** are parallel to OA and OC respectively and the unit vector **k** is vertically upwards. The position vectors of A and D are given by $\overrightarrow{OA} = 8\mathbf{i}$ and $\overrightarrow{OD} = 3\mathbf{i} + 10\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{AM}$ and $\overrightarrow{GM}$ in terms of **i**, **j** and **k**. [3]

$\overrightarrow{AM} = \dfrac{\overrightarrow{AF}}{2}$ $\quad \overrightarrow{AF} = \overrightarrow{AB} + \overrightarrow{BF}$

OABC is a square thus $\overrightarrow{AB} = 8j$

and $\overrightarrow{BF} = \overrightarrow{OD} = 3i + 10k$

$\overrightarrow{AF} = 3i + 8j + 10k$

$\overrightarrow{AM} = \frac{3}{2}i + 4j + 5k$

$\overrightarrow{GM} = \overrightarrow{GE} + \overrightarrow{EA} + \overrightarrow{AM}$

$\overrightarrow{GD} + \overrightarrow{DE}$ $\quad -\overrightarrow{OD}$

$\overrightarrow{GM} = -8j + 8i - 3i - 10k + \frac{3}{2}i + 4j + 5k$

$\overrightarrow{GM} = 6{\cdot}5i - 4j - 5k$

(ii) Use a scalar product to find angle *GMA* correct to the nearest degree. [4]

$\overrightarrow{GM} = 6.5i - 4j - 5k$

$\overrightarrow{AM} = 1.5i + 4j + 5k$

$\sqrt{(6.5)^2+(-4)^2+(-5)^2} \times \sqrt{(1.5)^2+(4)^2+(5)^2} \cos\theta = \begin{pmatrix} 6.5 \\ -4 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 1.5 \\ 4 \\ 5 \end{pmatrix}$

$60\cos\theta = -31.25$

$\theta = 121.4°$

8 **(a)** The third and fourth terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [3]

$a_n = a_1 r^{n-1}$

$a_3 = 48$, $a_4 = 32$

$S_\infty = \frac{a_1}{1-r}$

$r = \frac{32}{48} = \frac{2}{3}$

$48 = a_1 \times \left(\frac{2}{3}\right)^{3-1} \longrightarrow a_1 = 108$

$S_\infty = \frac{108}{1-\frac{2}{3}} = 324$

(b) Two schemes are proposed for increasing the amount of household waste that is recycled each week.

Scheme A is to increase the amount of waste recycled each month by 0.16 tonnes.

Scheme B is to increase the amount of waste recycled each month by 6% of the amount recycled in the previous month.

The proposal is to operate the scheme for a period of 24 months. The amount recycled in the first month is 2.5 tonnes.

For each scheme, find the total amount of waste that would be recycled over the 24-month period. [5]

Scheme A amount of recycled waste ↑ by 0·16 tonnes each month

$\underbrace{2.5, 2.66}_{0.16}, 2.82, \dots$ until $n = 24$

Use airthmetic sequence formula

$$S_n = \frac{n}{2}(2a_1 + d(n-1))$$

$$S_n = \frac{24}{2}(2(2.5) + 0.16(24-1))$$

104·16 tonnes of waste that is recycled

Scheme B amount of recycled waste ↑ by 6% each month

$2.5, 2.65, 2.809, \dots$ until $n = 24$

Use geometric sequence formula

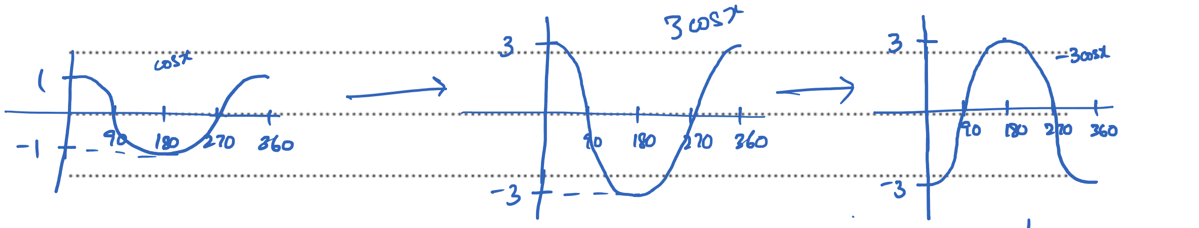
$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

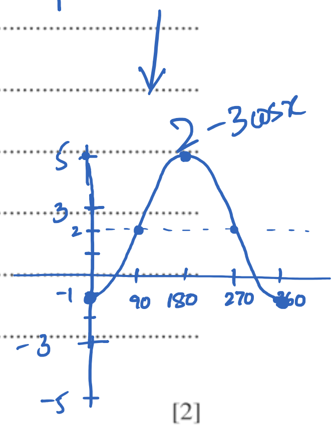
$$S_n = \frac{2.5(1 - 1.06^{24})}{1 - 1.06}$$

127·04 tonnes of waste that is recycled.

9 The function f is defined by $f(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant 2\pi$.

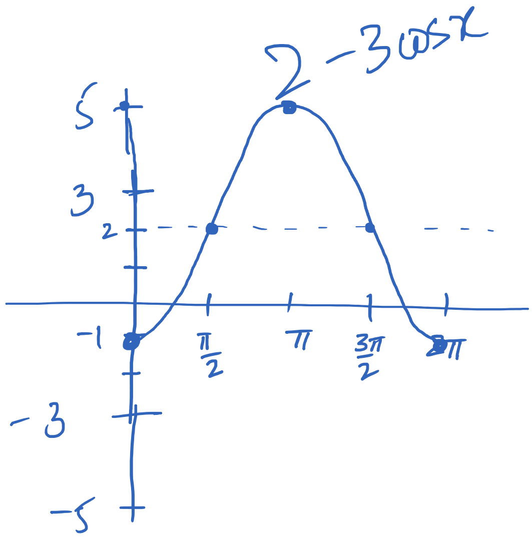
(i) State the range of f. [2]





$-1 \leq f(x) \leq 5$

(ii) Sketch the graph of $y = f(x)$. [2]



The function g is defined by $g(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant p$, where p is a constant.

(iii) State the largest value of p for which g has an inverse. [1]

$y = 2 - 3\cos x$

$\cos^{-1}\left(\frac{2-x}{3}\right) = y$

Domain of $f^{-1}(x) \longrightarrow -1 \leqslant x \leqslant 5$

Hence $0^\circ \leqslant f^{-1}(x) \leqslant 180^\circ$

$p = \pi$

(iv) For this value of p, find an expression for $g^{-1}(x)$. [2]

$g(x) = 2 - 3\cos x \quad 0 \leqslant x \leqslant \pi$

$\cos^{-1}\left(\frac{2-y}{3}\right) = x \longrightarrow g^{-1}(x) = \cos^{-1}\left(\frac{2-x}{3}\right)$

10 A curve for which $\frac{d^2y}{dx^2} = 2x - 5$ has a stationary point at (3, 6).

(i) Find the equation of the curve. [6]

Use $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \int 2x - 5 \longrightarrow x^2 - 5x + C$

$x^2 - 5x + C = 0$

$(3)^2 - 5(3) + C = 0 \longrightarrow C = 6$

$\frac{dy}{dx} = x^2 - 5x + 6$

$\int x^2 - 5x + 6 \longrightarrow y = \frac{x^3}{3} - \frac{5x^2}{2} + 6x + C$

Use (3, 6)

$6 = \frac{(3)^3}{3} - \frac{5}{2}(3)^2 + 6(3) + C$

$C = \frac{3}{2}$

$y = \frac{x^3}{3} - \frac{5}{2}x^2 + 6x + \frac{3}{2}$ //

(ii) Find the x-coordinate of the other stationary point on the curve. [1]

$\frac{dy}{dx} = 0 \longrightarrow x^2 - 5x + 6 = 0$

$x = 3, \; x = 2$

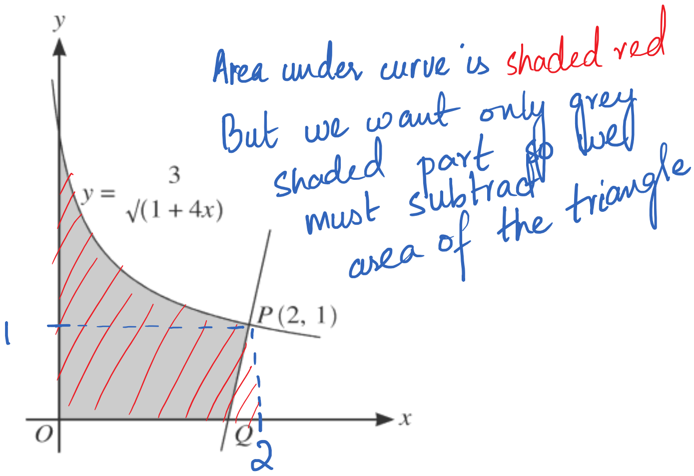
(iii) Determine the nature of each of the stationary points. [2]

Use $\frac{d^2y}{dx^2} = 2x - 5$

Use $x = 3$, $2(3) - 5 = 1$ thus $(3, 6)$ is a minimum point

Use $x = 2$, $2(2) - 5 = -1$ thus $\left(2, \frac{37}{6}\right)$ is a maximum point

11



The diagram shows part of the curve $y = \frac{3}{\sqrt{(1+4x)}}$ and a point $P\,(2, 1)$ lying on the curve. The normal to the curve at P intersects the x-axis at Q.

(i) Show that the x-coordinate of Q is $\frac{16}{9}$. [5]

For this, we must first have gradient of the normal so that we can get its equation

Gradient of curve at P x gradient of normal = −1

gradient of curve $= \frac{dy}{dx}$ $\quad \frac{dy}{dx} = -6(1+4x)^{-\frac{3}{2}}$

Gradient of curve at P $\longrightarrow -6(1+4(2))^{-\frac{3}{2}}$ is $-\frac{2}{9}$

Gradient of normal at P $\longrightarrow -1 \div -\frac{2}{9} = \frac{9}{2}$

Equation of normal using (2, 1) $\longrightarrow y - 1 = \frac{9}{2}(x-2)$

$y = \frac{9}{2}x - 8$

At Q, normal intersects x-axis

On x-axis, $y = 0$

$\frac{9}{2}x - 8 = 0 \longrightarrow x = \frac{16}{9}$

(ii) Find, showing all necessary working, the area of the shaded region. [6]

A of shaded region $= \int_0^2 \frac{3}{\sqrt{1+4x}}$ — A of triangle

$\int_0^2 3(1+4x)^{-\frac{1}{2}} = 3\left[2(1+4x)^{\frac{1}{2}} \times \frac{1}{4}\right]$

$\frac{3}{2}\left[\sqrt{1+4x}\right]_0^2 = 4.5 - 1.5 = 3$

A of triangle $= \frac{1}{2} \times \left(2 - \frac{16}{9}\right) \times 1 = \frac{1}{9}$

A of shaded region $= 3 - \frac{1}{9}$

$= \frac{26}{9}$

Additional Page

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